

Satellite Crash

Introduction: The Hubble Space Telescope (HST) zooms around the Earth once every 90 minutes (about 5 miles per second), and in August 2008, Hubble completed 100,000 orbits! Although the HST was not the first space telescope, is the one of the largest and most publicized scientific instrument around. Hubble is a collaboration project between NASA and the ESA (European Space Agency), and is one of NASA's "Great Observatories" (others include Compton Gamma Ray Observatory, Chandra X-Ray Observatory, and Spitzer Space Telescope). Anyone can apply for time on the telescope (you do not need to be affiliated with any academic institution or company), but it's a tight squeeze to get on the schedule.

Hubble's orbit zooms high in the upper atmosphere to steer clear of the obscuring haze of molecules in the sea of air. Hubble's orbit slowly decays over time and begins to spiral back into Earth until the astronauts bump it back up into a higher orbit.

Materials:

- Marble
- Paper
- Tape

Procedure:

1. Take your paper and roll it into a cone shape
2. Take a couple pieces of tape and secure the cone in place
3. Now take your marble and place it in the cone. Try to spin the cone at just the right speed so that the marble doesn't fly out, or fall back into the hole!
4. Once you've had some practice, see if you can maintain the marble's path at a specific distance from the center hole. This is just like changing a satellite's orbital radius from Earth!

The marble rolling around the cone is a good analogy for a satellite orbiting Earth. When a satellite is put into orbit, there are no big rockets on board that the satellite can use to maintain or change its orbit, so how does it stay at the same distance from Earth, and not fall in due to gravity?

Well the satellite is effectively constantly free falling around Earth, and since in space there is no air to create drag and slow it down, it maintains the same speed, and thus the same orbital radius.

Using the Hubble Space Telescope as an example, let's analyze the motion of a satellite orbiting Earth. In this case we can assume that the Earth has a much larger mass than the satellite, which makes things easier. The equation for the orbital velocity (**v**) of the satellite is given as follows:

$$v = \sqrt{\frac{GM}{r}}$$

Where **v** is the orbital velocity, **G** is the Gravitational Constant ($6.67 \times 10^{-11} \text{ m/kg}^2$), **M** is the mass of the Earth, and **r** is the radius from the center of the Earth to the orbit of the satellite.

So let's calculate the height above Earth's surface that Hubble orbits at. We know it's traveling around 5 miles per second (it's actually traveling 7,500 meters per second). We also know the mass of the Earth is $5.97 \times 10^{24} \text{ kg}$, so what is **r**?

$$7,500 \text{ m/s} = (6.67 \times 10^{-11} \text{ m/kg}^2 * 5.97 \times 10^{24} \text{ kg} / r)^{1/2}$$

$$56,250,000 \text{ m}^2/\text{s}^2 = 3.98 \times 10^{14} \text{ m/kg} * r$$

$$r = 7.08 \times 10^6 \text{ meters from the center of Earth}$$

Earth has a radius of about 6,371,000 m, so Hubble is orbiting at a height of about 700 km above the surface of the Earth.

Problems:

1. Imagine a satellite is orbiting very far from Earth, with $r = 550,000$ km. How fast will the satellite be orbiting if it is to maintain a circular orbit?
2. If we assume the Earth's mass is sufficiently smaller than the Sun's mass, how far from the center of the Sun are we if we orbit at 30 km/s? (Hint: the mass of the Sun is 1.989×10^{30} kg)
3. How far from Earth would a satellite be if it were orbiting at 12,000 m/s?

Answers:

1. 850 m/s
2. 1.47×10^{11} m or 147 million km!
3. It's impossible! If no other forces are acting on the satellite, they would have to be below the surface of the Earth!