Isaac Newton

Chapter 4

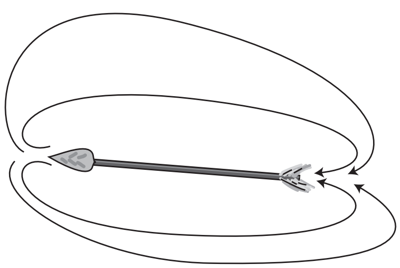
Force and Motion

> If I have seen farther than others, it is because I have stood on the shoulders of giants.
>
> *Newton, referring to Galileo*

Even as great and skeptical a genius as Galileo was unable to make much progress on the causes of motion. It was not until a generation later that Isaac Newton (1642-1727) was able to attack the problem successfully. In many ways, Newton's personality was the opposite of Galileo's. Where Galileo agressively publicized his ideas, Newton had to be coaxed by his friends into publishing a book on his physical discoveries. Where Galileo's writing had been popular and dramatic, Newton originated the stilted, impersonal style that most people think is standard for scientific writing. (Scientific journals today encourage a less ponderous style, and papers are often written in the first person.) Galileo's talent for arousing animosity among the rich and powerful was matched by Newton's skill at making himself a popular visitor at court. Galileo narrowly escaped being burned at the stake, while Newton had the good fortune of being on the winning side of the revolution that replaced King James II with William and Mary of Orange, leading to a lucrative post running the English royal mint.

Newton discovered the relationship between force and motion, and revolutionized our view of the universe by showing that the same physical laws applied to all matter, whether living or nonliving, on or off of our planet's surface. His book on force and motion, the **Mathematical Principles of Natural Philosophy**, was uncontradicted by experiment for 200 years, but his other main work, **Optics**, was on the wrong track, asserting that light was composed of particles rather than waves. Newton was also an avid alchemist, a fact that modern scientists would like to forget.

4.1 Force



a / Aristotle said motion had to be caused by a force. To explain why an arrow kept flying after the bowstring was no longer pushing on it, he said the air rushed around behind the arrow and pushed it forward. We know this is wrong, because an arrow shot in a vacuum chamber does not instantly drop to the floor as it leaves the bow. Galileo and Newton realized that a force would only be needed to change the arrow's motion, not to make its motion continue.

We need only explain changes in motion, not motion itself.

So far you've studied the measurement of motion in some detail, but not the reasons why a certain object would move in a certain way. This chapter deals with the "why" questions. Aristotle's ideas about the causes of motion were completely wrong, just like all his other ideas about physical science, but it will be instructive to start with them, because they amount to a road map of modern students' incorrect preconceptions.

Aristotle thought he needed to explain both why motion occurs and why motion might change. Newton inherited from Galileo the important counter-Aristotelian idea that motion needs no explanation, that it is only *changes* in motion that require a physical cause. Aristotle's needlessly complex system gave three reasons for motion:

> Natural motion, such as falling, came from the tendency of objects to go to their "natural" place, on the ground, and come to rest.
>
> Voluntary motion was the type of motion exhibited by animals, which moved because they chose to.
>
> Forced motion occurred when an object was acted on by some other object that made it move.

Motion changes due to an interaction between two objects.

In the Aristotelian theory, natural motion and voluntary motion are one-sided phenomena: the object causes its own motion. Forced motion is supposed to be a two-sided phenomenon, because one object imposes its "commands" on another. Where Aristotle conceived of some of the phenomena of motion as one-sided and others as two-sided, Newton realized that a change in motion was always a two-sided relationship of a force acting between two physical objects.

The one-sided "natural motion" description of falling makes a crucial omission. The acceleration of a falling object is not caused by its own "natural" tendencies but by an attractive force between it and the planet Earth. Moon rocks brought back to our planet do not "want" to fly back up to the moon because the moon is their "natural" place. They fall to the floor when you drop them, just like our homegrown rocks. As we'll discuss in more detail later in this course, gravitational forces are simply an attraction that occurs between any two physical objects. Minute gravitational forces can even be measured between human-scale objects in the laboratory.

The idea of natural motion also explains incorrectly why things come to rest. A basketball rolling across a beach slows to a stop because it is interacting with the sand via a frictional force, not because of its own desire to be at rest. If it was on a frictionless surface, it would never slow down. Many of Aristotle's mistakes stemmed from his failure to recognize friction as a force.

b / "Our eyes receive blue light reflected from this painting because Monet wanted to represent water with the color blue." This is a valid statement at one level of explanation, but physics works at the physical level of explanation, in which blue light gets to your eyes because it is reflected by blue pigments in the paint.

The concept of voluntary motion is equally flawed. You may have been a little uneasy about it from the start, because it assumes a clear distinction between living and nonliving things. Today, however, we are used to having the human body likened to a complex machine. In the modern world-view, the border between the living and the inanimate is a fuzzy no-man's land inhabited by viruses, prions, and silicon chips. Furthermore, Aristotle's statement that you can take a step forward "because you choose to" inappropriately mixes two levels of explanation. At the physical level of explanation, the reason your body steps forward is because of a frictional force acting between your foot and the floor. If the floor was covered with a puddle of oil, no amount of "choosing to" would enable you to take a graceful stride forward.

Forces can all be measured on the same numerical scale.

In the Aristotelian-scholastic tradition, the description of motion as natural, voluntary, or forced was only the broadest level of classification, like splitting animals into birds, reptiles, mammals, and amphibians. There might be thousands of types of motion, each of which would follow its own rules. Newton's realization that all changes in motion were caused by two-sided interactions made it seem that the phenomena might have more in common than had been apparent. In the Newtonian description, there is only one cause for a change in motion, which we call force. Forces may be of different types, but they all produce changes in motion according to the same rules. Any acceleration that can be produced by a magnetic force can equally well be produced by an appropriately controlled stream of water. We can speak of two forces as being equal if they produce the same change in motion when applied in the same situation, which means that they pushed or pulled equally hard in the same direction.

The idea of a numerical scale of force and the newton unit were introduced in chapter 0. To recapitulate briefly, a force is when a pair of objects push or pull on each other, and one newton is the force required to accelerate a 1-kg object from rest to a speed of 1 m/s in 1 second.

More than one force on an object

As if we hadn't kicked poor Aristotle around sufficiently, his theory has another important flaw, which is important to discuss because it corresponds to an extremely common student misconception. Aristotle conceived of forced motion as a relationship in which one object was the boss and the other "followed orders." It therefore would only make sense for an object to experience one force at a time, because an object couldn't follow orders from two sources at once. In the Newtonian theory, forces are numbers, not orders, and if more than one force acts on an object at once, the result is found by adding up all the forces. It is unfortunate that the use of the English word "force" has become standard, because to many people it suggests that you are "forcing" an object to do something. The force of the earth's gravity cannot "force" a boat to sink, because there are other forces acting on the boat. Adding them up gives a total of zero, so the boat accelerates neither up nor down.

Objects can exert forces on each other at a distance.

Aristotle declared that forces could only act between objects that were touching, probably because he wished to avoid the type of occult speculation that attributed physical phenomena to the influence of a distant and invisible pantheon of gods. He was wrong, however, as you can observe when a magnet leaps onto your refrigerator or when the planet earth exerts gravitational forces on objects that are in the air. Some types of forces, such as friction, only operate between objects in contact, and are called contact forces. Magnetism, on the other hand, is an example of a noncontact force. Although the magnetic force gets stronger when the magnet is closer to your refrigerator, touching is not required.

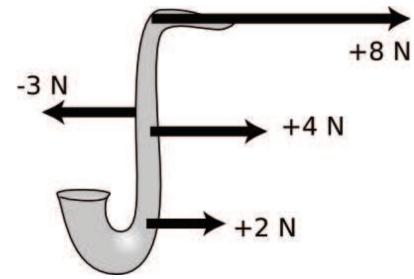
Weight

In physics, an object's weight, F_W, is defined as the earth's gravitational force on it. The SI unit of weight is therefore the

Newton. People commonly refer to the kilogram as a unit of weight, but the kilogram is a unit of mass, not weight. Note that an object's weight is not a fixed property of that object. Objects weigh more in some places than in others, depending on the local strength of gravity. It is their mass that always stays the same. A baseball pitcher who can throw a 90-mile-per-hour fastball on earth would not be able to throw any faster on the moon, because the ball's inertia would still be the same.

Positive and negative signs of force

We'll start by considering only cases of one-dimensional center-of-mass motion in which all the forces are parallel to the direction of motion, i.e., either directly forward or backward. In one dimension, plus and minus signs can be used to indicate directions of forces, as shown in figure c. We can then refer generically to addition of forces, rather than having to speak sometimes of addition and sometimes of subtraction. We add the forces shown in the figure and get 11 N. In general, we should choose a one-dimensional coordinate system with its x axis parallel the direction of motion. Forces that point along the positive x axis are positive, and forces in the opposite direction are negative. Forces that are not directly along the x axis cannot be immediately incorporated into this scheme, but that's OK, because we're avoiding those cases for now.



c / Forces are applied to a saxophone. In this example, positive signs have been used consistently for forces to the right, and negative signs for forces to the left. (The forces are being applied to different places on the saxophone, but the numerical value of a force carries no information about that.)

Discussion Questions

A In chapter 0, I defined 1 N as the force that would accelerate a 1-kg mass from rest to 1 m/s in 1 s. Anticipating the following section, you might guess that 2 N could be defined as the force that would accelerate the same mass to twice the speed, or twice the mass to the same speed. Is there an easier way to define 2 N based on the definition of 1 N?

4.2 Newton's First Law

We are now prepared to make a more powerful restatement of the principle of inertia.

Newton's first law
If the total force on an object is zero, its center of mass continues in the same state of motion.

In other words, an object initially at rest is predicted to remain at rest if the total force on it is zero, and an object in motion remains in motion with the same velocity in the same direction. The converse of Newton's first law is also true: if we observe an object moving with constant velocity along a straight line, then the total force on it must be zero.

In a future physics course or in another textbook, you may encounter the term "net force," which is simply a synonym for total force.

What happens if the total force on an object is not zero? It accelerates. Numerical prediction of the resulting acceleration is the topic of Newton's second law, which we'll discuss in the following section.

This is the first of Newton's three laws of motion. It is not important to memorize which of Newton's three laws are numbers one, two, and three. If a future physics teacher asks you something like, "Which of Newton's laws are you thinking of," a perfectly acceptable answer is "The one about constant velocity when there's zero total force." The concepts are more important than any specific formulation of them. Newton wrote in Latin, and I am not aware of any modern textbook that uses a verbatim translation of his statement of the laws of motion. Clear writing was not in vogue in Newton's day, and he formulated his three laws in terms of a concept now called momentum, only later relating it to the concept of force. Nearly all modern texts, including this one, start with force and do momentum later.

An elevator *example 1*

▷ An elevator has a weight of 5000 N. Compare the forces that the cable must exert to raise it at constant velocity, lower it at constant velocity, and just keep it hanging.

▷ In all three cases the cable must pull up with a force of exactly 5000 N. Most people think you'd need at least a little more than 5000 N to make it go up, and a little less than 5000 N to let it down, but that's incorrect. Extra force from the cable is only necessary for speeding the car up when it starts going up or slowing it down when it finishes going down. Decreased force is needed to speed the car up when it gets going down and to slow it down when it finishes going up. But when the elevator is cruising at constant velocity, Newton's first law says that you just need to cancel the force of the earth's gravity.

To many students, the statement in the example that the cable's upward force "cancels" the earth's downward gravitational force implies that there has been a contest, and the cable's force has won, vanquishing the earth's gravitational force and making it disappear. That is incorrect. Both forces continue to exist, but because they add up numerically to zero, the elevator has no center-of-mass acceleration. We know that both forces continue to exist because they both have side-effects other than their effects on the car's center-of-mass motion. The force acting between the cable and the car continues to produce tension in the cable and keep the cable taut. The earth's gravitational force continues to keep the passengers (whom we are considering as part of the elevator-object) stuck to the floor and to produce internal stresses in the walls of the car, which must hold up the floor.

Terminal velocity for falling objects *example 2*

▷ An object like a feather that is not dense or streamlined does not fall with constant acceleration, because air resistance is nonnegligible. In fact, its acceleration tapers off to nearly zero within a fraction of a second, and the feather finishes dropping at constant speed (known as its terminal velocity). Why does this happen?

▷ Newton's first law tells us that the total force on the feather must have been reduced to nearly zero after a short time. There are two forces acting on the feather: a downward gravitational force from the planet earth, and an upward frictional force from the air. As the feather speeds up, the air friction becomes stronger and stronger, and eventually it cancels out the earth's gravitational force, so the feather just continues with constant velocity without speeding up any more.

The situation for a skydiver is exactly analogous. It's just that the skydiver experiences perhaps a million times more gravitational force than the feather, and it is not until she is falling very fast that the force of air friction becomes as strong as the gravitational force. It takes her several seconds to reach terminal velocity, which is on the order of a hundred miles per hour.

More general combinations of forces

It is too constraining to restrict our attention to cases where all the forces lie along the line of the center of mass's motion. For one thing, we can't analyze any case of horizontal motion, since any object on earth will be subject to a vertical gravitational force! For instance, when you are driving your car down a straight road, there are both horizontal forces and vertical forces. However, the vertical forces have no effect on the center of mass motion, because the road's upward force simply counteracts the earth's downward gravitational force and keeps the car from sinking into the ground.

Later in the book we'll deal with the most general case of many forces acting on an object at any angles, using the mathematical technique of vector addition, but the following slight generalization of Newton's first law allows us to analyze a great many cases of interest:

Suppose that an object has two sets of forces acting on it, one set along the line of the object's initial motion and another set perpendicular to the first set. If both sets of forces cancel, then the object's center of mass continues in the same state of motion.

A passenger riding the subway *example 3*

▷ Describe the forces acting on a person standing in a subway train that is cruising at constant velocity.

▷ No force is necessary to keep the person moving relative to the ground. He will not be swept to the back of the train if the floor is slippery. There are two vertical forces on him, the earth's downward gravitational force and the floor's upward force, which cancel. There are no horizontal forces on him at all, so of course the total horizontal force is zero.

Forces on a sailboat *example 4*

▷ If a sailboat is cruising at constant velocity with the wind coming from directly behind it, what must be true about the forces acting on it?

▷ The forces acting on the boat must be canceling each other out. The boat is not sinking or leaping into the air, so evidently the vertical forces are canceling out. The vertical forces are the downward gravitational force exerted by the planet earth and an upward force from the water.

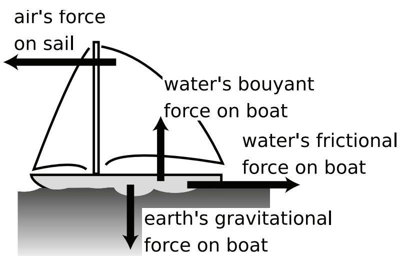
The air is making a forward force on the sail, and if the boat is not accelerating horizontally then the water's backward frictional force must be canceling it out.

Contrary to Aristotle, more force is not needed in order to maintain a higher speed. Zero total force is always needed to maintain constant velocity. Consider the following made-up numbers:

	boat moving at a low, constant velocity	boat moving at a high, constant velocity
forward force of the wind on the sail . . .	10,000 N	20,000 N
backward force of the water on the hull . . .	$-10,000$ N	$-20,000$ N
total force on the boat . . .	0 N	0 N

The faster boat still has zero total force on it. The forward force on it is greater, and the backward force smaller (more negative), but that's irrelevant because Newton's first law has to do with the total force, not the individual forces.

This example is quite analogous to the one about terminal velocity of falling objects, since there is a frictional force that increases with speed. After casting off from the dock and raising the sail, the boat will accelerate briefly, and then reach its terminal velocity, at which the water's frictional force has become as great as the wind's force on the sail.



d / Example 4.

A car crash *example 5*

▷ If you drive your car into a brick wall, what is the mysterious force that slams your face into the steering wheel?

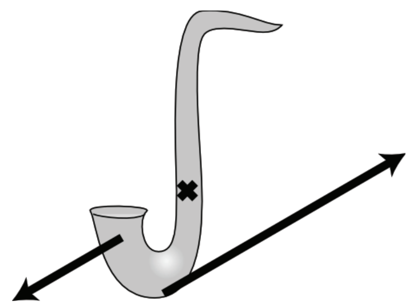
▷ Your surgeon has taken physics, so she is not going to believe your claim that a mysterious force is to blame. She knows that your face was just following Newton's first law. Immediately after your car hit the wall, the only forces acting on your head were the same canceling-out forces that had existed previously: the earth's downward gravitational force and the upward force from your neck. There were no forward or backward forces on your head, but the car did experience a backward force from the wall, so the car slowed down and your face caught up.

Discussion Questions

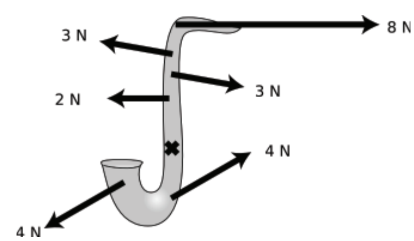
A Newton said that objects continue moving if no forces are acting on them, but his predecessor Aristotle said that a force was necessary to keep an object moving. Why does Aristotle's theory seem more plausible, even though we now believe it to be wrong? What insight was Aristotle missing about the reason why things seem to slow down naturally? Give an example.

B In the figure what would have to be true about the saxophone's initial motion if the forces shown were to result in continued one-dimensional motion of its center of mass?

C This figure requires an ever further generalization of the preceding discussion. After studying the forces, what does your physical intuition tell you will happen? Can you state in words how to generalize the conditions for one-dimensional motion to include situations like this one?



Discussion question B.



Discussion question C.

4.3 Newton's Second Law

What about cases where the total force on an object is not zero, so that Newton's first law doesn't apply? The object will have an acceleration. The way we've defined positive and negative signs of force and acceleration guarantees that positive forces produce positive accelerations, and likewise for negative values. How much acceleration will it have? It will clearly depend on both the object's mass and on the amount of force.

Experiments with any particular object show that its acceleration is directly proportional to the total force applied to it. This may seem wrong, since we know of many cases where small amounts of force fail to move an object at all, and larger forces get it going. This apparent failure of proportionality actually results from forgetting that there is a frictional force in addition to the force we apply to move the object. The object's acceleration is exactly proportional to the total force on it, not to any individual force on it. In the absence of friction, even a very tiny force can slowly change the velocity of a very massive object.

Experiments also show that the acceleration is inversely proportional to the object's mass, and combining these two proportionalities gives the following way of predicting the acceleration of any object:

Newton's second law

$$a = F_{total}/m \quad ,$$

where

m is an object's mass
F_{total} is the sum of the forces acting on it, and
a is the acceleration of the object's center of mass.

We are presently restricted to the case where the forces of interest are parallel to the direction of motion.

An accelerating bus *example 6*

▷ A VW bus with a mass of 2000 kg accelerates from 0 to 25 m/s (freeway speed) in 34 s. Assuming the acceleration is constant, what is the total force on the bus?

▷ We solve Newton's second law for $F_{total} = ma$, and substitute $\Delta v/\Delta t$ for a, giving

$$\begin{aligned} F_{total} &= m\Delta v/\Delta t \\ &= (2000\ \text{kg})(25\ \text{m/s} - 0\ \text{m/s})/(34\ \text{s}) \\ &= 1.5\ \text{kN} \quad . \end{aligned}$$

A generalization

As with the first law, the second law can be easily generalized to include a much larger class of interesting situations:

> Suppose an object is being acted on by two sets of forces, one set lying along the object's initial direction of motion and another set acting along a perpendicular line. If the forces perpendicular to the initial direction of motion cancel out, then the object accelerates along its original line of motion according to $a = F_{total}/m$.

The relationship between mass and weight

Mass is different from weight, but they're related. An apple's mass tells us how hard it is to change its motion. Its weight measures the strength of the gravitational attraction between the apple and the planet earth. The apple's weight is less on the moon, but its mass is the same. Astronauts assembling the International Space Station in zero gravity cannot just pitch massive modules back and forth with their bare hands; the modules are weightless, but not massless.

We have already seen the experimental evidence that when weight (the force of the earth's gravity) is the only force acting on an object, its acceleration equals the constant g, and g depends on where you are on the surface of the earth, but not on the mass of the object. Applying Newton's second law then allows us to calculate the magnitude of the gravitational force on any object in terms of its mass:

$$|F_W| = mg \qquad .$$

(The equation only gives the magnitude, i.e. the absolute value, of F_W, because we're defining g as a positive number, so it equals the absolute value of a falling object's acceleration.)

▷ *Solved problem: Decelerating a car* *page 142, problem 7*

Weight and mass *example 7*

▷ Figure f shows masses of one and two kilograms hung from a spring scale, which measures force in units of newtons. Explain the readings.

▷ Let's start with the single kilogram. It's not accelerating, so evidently the total force on it is zero: the spring scale's upward force on it is canceling out the earth's downward gravitational force. The spring scale tells us how much force it is being obliged to supply, but since the two forces are equal in strength, the spring scale's reading can also be interpreted as measuring the strength of the gravitational force, i.e., the weight of the one-kilogram mass. The weight of a one-kilogram mass should be

$$\begin{aligned} F_W &= mg \\ &= (1.0\ \text{kg})(9.8\ \text{m/s}^2) = 9.8\ \text{N} \qquad , \end{aligned}$$

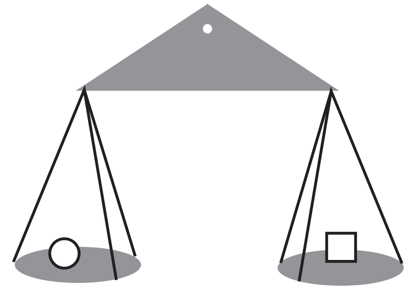
and that's indeed the reading on the spring scale.

Similarly for the two-kilogram mass, we have

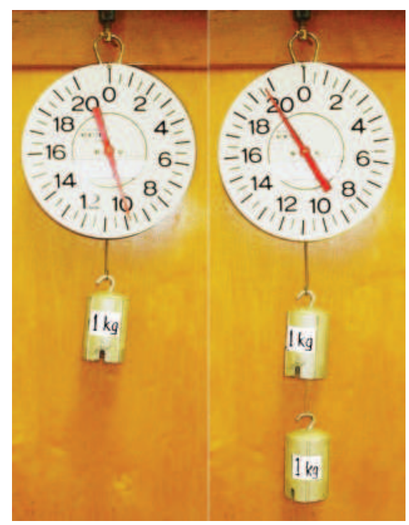
$$\begin{aligned} F_W &= mg \\ &= (2.0\ \text{kg})(9.8\ \text{m/s}^2) = 19.6\ \text{N} \qquad . \end{aligned}$$

Calculating terminal velocity *example 8*

▷ Experiments show that the force of air friction on a falling object such as a skydiver or a feather can be approximated fairly well with the equation $|F_{air}| = c\rho Av^2$, where c is a constant, ρ is the density of the air, A is the cross-sectional area of the object as seen from below, and v is the object's velocity. Predict the object's terminal velocity, i.e., the final velocity it reaches after a long time.



e / A simple double-pan balance works by comparing the weight forces exerted by the earth on the contents of the two pans. Since the two pans are at almost the same location on the earth's surface, the value of g is essentially the same for each one, and equality of weight therefore also implies equality of mass.



f / Example 7.

▷ As the object accelerates, its greater v causes the upward force of the air to increase until finally the gravitational force and the force of air friction cancel out, after which the object continues at constant velocity. We choose a coordinate system in which positive is up, so that the gravitational force is negative and the force of air friction is positive. We want to find the velocity at which

$$F_{air} + F_W = 0 \quad , \quad i.e.,$$
$$c\rho A v^2 - mg = 0 \quad .$$

Solving for v gives

$$v_{terminal} = \sqrt{\frac{mg}{c\rho A}}$$

self-check A

It is important to get into the habit of interpreting equations. This may be difficult at first, but eventually you will get used to this kind of reasoning.

(1) Interpret the equation $v_{terminal} = \sqrt{mg/c\rho A}$ in the case of $\rho=0$.

(2) How would the terminal velocity of a 4-cm steel ball compare to that of a 1-cm ball?

(3) In addition to teasing out the *mathematical* meaning of an equation, we also have to be able to place it in its *physical* context. How generally important is this equation? ▷ Answer, p. 272

x (m)	t (s)
10	1.84
20	2.86
30	3.80
40	4.67
50	5.53
60	6.38
70	7.23
80	8.10
90	8.96
100	9.83

g / Discussion question D.

Discussion Questions

A Show that the Newton can be reexpressed in terms of the three basic mks units as the combination $\text{kg}\cdot\text{m}/\text{s}^2$.

B What is wrong with the following statements?

(1) “g is the force of gravity.”

(2) “Mass is a measure of how much space something takes up.”

C Criticize the following incorrect statement:

“If an object is at rest and the total force on it is zero, it stays at rest. There can also be cases where an object is moving and keeps on moving without having any total force on it, but that can only happen when there’s no friction, like in outer space.”

D Table g gives laser timing data for Ben Johnson’s 100 m dash at the 1987 World Championship in Rome. (His world record was later revoked because he tested positive for steroids.) How does the total force on him change over the duration of the race?

4.4 What Force Is Not

Violin teachers have to endure their beginning students' screeching. A frown appears on the woodwind teacher's face as she watches her student take a breath with an expansion of his ribcage but none in his belly. What makes physics teachers cringe is their students' verbal statements about forces. Below I have listed several dicta about what force is not.

Force is not a property of one object.

A great many of students' incorrect descriptions of forces could be cured by keeping in mind that a force is an interaction of two objects, not a property of one object.

Incorrect statement: "That magnet has a lot of force."

✗If the magnet is one millimeter away from a steel ball bearing, they may exert a very strong attraction on each other, but if they were a meter apart, the force would be virtually undetectable. The magnet's strength can be rated using certain electrical units ($\text{ampere} - \text{meters}^2$), but not in units of force.

Force is not a measure of an object's motion.

If force is not a property of a single object, then it cannot be used as a measure of the object's motion.

Incorrect statement: "The freight train rumbled down the tracks with awesome force."

✗Force is not a measure of motion. If the freight train collides with a stalled cement truck, then some awesome forces will occur, but if it hits a fly the force will be small.

Force is not energy.

There are two main approaches to understanding the motion of objects, one based on force and one on a different concept, called energy. The SI unit of energy is the Joule, but you are probably more familiar with the calorie, used for measuring food's energy, and the kilowatt-hour, the unit the electric company uses for billing you. Physics students' previous familiarity with calories and kilowatt-hours is matched by their universal unfamiliarity with measuring forces in units of Newtons, but the precise operational definitions of the energy concepts are more complex than those of the force concepts, and textbooks, including this one, almost universally place the force description of physics before the energy description. During the long period after the introduction of force and before the careful definition of energy, students are therefore vulnerable to situations in which, without realizing it, they are imputing the properties of energy to phenomena of force.

Incorrect statement: "How can my chair be making an upward force on my rear end? It has no power!"

✗Power is a concept related to energy, e.g., a 100-watt lightbulb uses up 100 joules per second of energy. When you sit in a chair, no energy is used up, so forces can exist between you and the chair without any need for a source of power.

Force is not stored or used up.

Because energy can be stored and used up, people think force also can be stored or used up.

Incorrect statement: "If you don't fill up your tank with gas, you'll run out of force."

✗Energy is what you'll run out of, not force.

Forces need not be exerted by living things or machines.

Transforming energy from one form into another usually requires some kind of living or mechanical mechanism. The concept is not applicable to forces, which are an interaction between objects, not a thing to be transferred or transformed.

Incorrect statement: "How can a wooden bench be making an upward force on my rear end? It doesn't have any springs or anything inside it."

✗No springs or other internal mechanisms are required. If the bench didn't make any force on you, you would obey Newton's second law and fall through it. Evidently it does make a force on you!

A force is the direct cause of a change in motion.

I can click a remote control to make my garage door change from being at rest to being in motion. My finger's force on the button, however, was not the force that acted on the door. When we speak of a force on an object in physics, we are talking about a force that acts directly. Similarly, when you pull a reluctant dog along by its leash, the leash and the dog are making forces on each other, not your hand and the dog. The dog is not even touching your hand.

self-check B
Which of the following things can be correctly described in terms of force?

(1) A nuclear submarine is charging ahead at full steam.

(2) A nuclear submarine's propellers spin in the water.

(3) A nuclear submarine needs to refuel its reactor periodically. ▷ Answer, p. 272

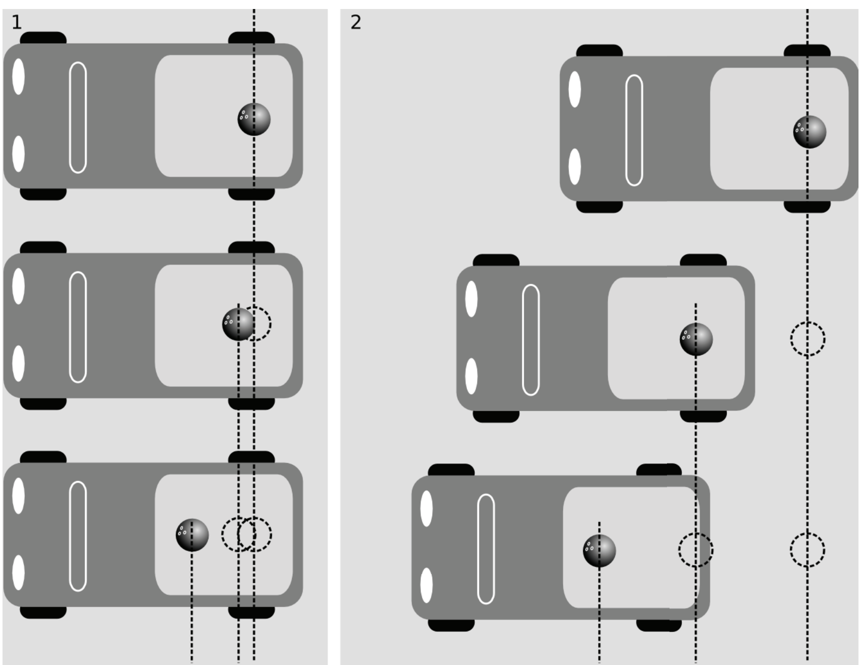
Discussion Questions

A Criticize the following incorrect statement: "If you shove a book across a table, friction takes away more and more of its force, until finally it stops."

B You hit a tennis ball against a wall. Explain any and all incorrect ideas in the following description of the physics involved: "The ball gets some force from you when you hit it, and when it hits the wall, it loses part of that force, so it doesn't bounce back as fast. The muscles in your arm are the only things that a force can come from."

4.5 Inertial and Noninertial Frames of Reference

One day, you're driving down the street in your pickup truck, on your way to deliver a bowling ball. The ball is in the back of the truck, enjoying its little jaunt and taking in the fresh air and sunshine. Then you have to slow down because a stop sign is coming up. As you brake, you glance in your rearview mirror, and see your trusty companion accelerating toward you. Did some mysterious force push it forward? No, it only seems that way because you and the car are slowing down. The ball is faithfully obeying Newton's first law, and as it continues at constant velocity it gets ahead relative to the slowing truck. No forces are acting on it (other than the same canceling-out vertical forces that were always acting on it).[1] The ball only appeared to violate Newton's first law because there was something wrong with your frame of reference, which was based on the truck.



h / 1. In a frame of reference that moves with the truck, the bowling ball appears to violate Newton's first law by accelerating despite having no horizontal forces on it. 2. In an inertial frame of reference, which the surface of the earth approximately is, the bowling ball obeys Newton's first law. It moves equal distances in equal time intervals, i.e., maintains constant velocity. In this frame of reference, it is the truck that appears to have a change in velocity, which makes sense, since the road is making a horizontal force on it.

How, then, are we to tell in which frames of reference Newton's laws are valid? It's no good to say that we should avoid moving frames of reference, because there is no such thing as absolute rest or absolute motion. All frames can be considered as being either at rest or in motion. According to an observer in India, the strip mall that constituted the frame of reference in panel (b) of the figure was moving along with the earth's rotation at hundreds of miles per hour.

[1] Let's assume for simplicity that there is no friction.

The reason why Newton's laws fail in the truck's frame of reference is not because the truck is *moving* but because it is *accelerating*. (Recall that physicists use the word to refer either to speeding up or slowing down.) Newton's laws were working just fine in the moving truck's frame of reference as long as the truck was moving at constant velocity. It was only when its speed changed that there was a problem. How, then, are we to tell which frames are accelerating and which are not? What if you claim that your truck is not accelerating, and the sidewalk, the asphalt, and the Burger King are accelerating? The way to settle such a dispute is to examine the motion of some object, such as the bowling ball, which we know has zero total force on it. Any frame of reference in which the ball appears to obey Newton's first law is then a valid frame of reference, and to an observer in that frame, Mr. Newton assures us that all the other objects in the universe will obey his laws of motion, not just the ball.

Valid frames of reference, in which Newton's laws are obeyed, are called inertial frames of reference. Frames of reference that are not inertial are called noninertial frames. In those frames, objects violate the principle of inertia and Newton's first law. While the truck was moving at constant velocity, both it and the sidewalk were valid inertial frames. The truck became an invalid frame of reference when it began changing its velocity.

You usually assume the ground under your feet is a perfectly inertial frame of reference, and we made that assumption above. It isn't perfectly inertial, however. Its motion through space is quite complicated, being composed of a part due to the earth's daily rotation around its own axis, the monthly wobble of the planet caused by the moon's gravity, and the rotation of the earth around the sun. Since the accelerations involved are numerically small, the earth is approximately a valid inertial frame.

Noninertial frames are avoided whenever possible, and we will seldom, if ever, have occasion to use them in this course. Sometimes, however, a noninertial frame can be convenient. Naval gunners, for instance, get all their data from radars, human eyeballs, and other detection systems that are moving along with the earth's surface. Since their guns have ranges of many miles, the small discrepancies between their shells' actual accelerations and the accelerations predicted by Newton's second law can have effects that accumulate and become significant. In order to kill the people they want to kill, they have to add small corrections onto the equation $a = F_{total}/m$. Doing their calculations in an inertial frame would allow them to use the usual form of Newton's second law, but they would have to convert all their data into a different frame of reference, which would require cumbersome calculations.

Discussion Question

A If an object has a linear $x - t$ graph in a certain inertial frame, what is the effect on the graph if we change to a coordinate system with a different origin? What is the effect if we keep the same origin but reverse the positive direction of the x axis? How about an inertial frame moving alongside the object? What if we describe the object's motion in a noninertial frame?

Summary

Selected Vocabulary

weight	the force of gravity on an object, equal to mg
inertial frame . .	a frame of reference that is not accelerating, one in which Newton's first law is true
noninertial frame	an accelerating frame of reference, in which Newton's first law is violated

Notation

F_W	weight

Other Terminology and Notation

net force	another way of saying "total force"

Summary

Newton's first law of motion states that if all the forces on an object cancel each other out, then the object continues in the same state of motion. This is essentially a more refined version of Galileo's principle of inertia, which did not refer to a numerical scale of force.

Newton's second law of motion allows the prediction of an object's acceleration given its mass and the total force on it, $a_{cm} = F_{total}/m$. This is only the one-dimensional version of the law; the full-three dimensional treatment will come in chapter 8, Vectors. Without the vector techniques, we can still say that the situation remains unchanged by including an additional set of vectors that cancel among themselves, even if they are not in the direction of motion.

Newton's laws of motion are only true in frames of reference that are not accelerating, known as inertial frames.

Exploring Further

Isaac Newton: The Last Sorcerer, Michael White. An excellent biography of Newton that brings us closer to the real man.

Problems

Key

- √ A computerized answer check is available online.
- ∫ A problem that requires calculus.
- ⋆ A difficult problem.

1 An object is observed to be moving at constant speed in a certain direction. Can you conclude that no forces are acting on it? Explain. [Based on a problem by Serway and Faughn.]

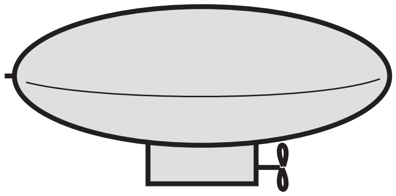
2 A car is normally capable of an acceleration of 3 m/s^2. If it is towing a trailer with half as much mass as the car itself, what acceleration can it achieve? [Based on a problem from PSSC Physics.]

3 (a) Let T be the maximum tension that an elevator's cable can withstand without breaking, i.e., the maximum force it can exert. If the motor is programmed to give the car an acceleration a, what is the maximum mass that the car can have, including passengers, if the cable is not to break? √
(b) Interpret the equation you derived in the special cases of $a = 0$ and of a downward acceleration of magnitude g.

("Interpret" means to analyze the behavior of the equation, and connect that to reality, as in the self-check on page 134.)

4 A helicopter of mass m is taking off vertically. The only forces acting on it are the earth's gravitational force and the force, F_{air}, of the air pushing up on the propeller blades.
(a) If the helicopter lifts off at $t = 0$, what is its vertical speed at time t?
(b) Plug numbers into your equation from part a, using $m = 2300$ kg, $F_{air} = 27000$ N, and $t = 4.0$ s.

5 In the 1964 Olympics in Tokyo, the best men's high jump was 2.18 m. Four years later in Mexico City, the gold medal in the same event was for a jump of 2.24 m. Because of Mexico City's altitude (2400 m), the acceleration of gravity there is lower than that in Tokyo by about 0.01 m/s^2. Suppose a high-jumper has a mass of 72 kg.
(a) Compare his mass and weight in the two locations.
(b) Assume that he is able to jump with the same initial vertical velocity in both locations, and that all other conditions are the same except for gravity. How much higher should he be able to jump in Mexico City? √
(Actually, the reason for the big change between '64 and '68 was the introduction of the "Fosbury flop.") ⋆



Problem 6.

6 A blimp is initially at rest, hovering, when at $t = 0$ the pilot turns on the motor of the propeller. The motor cannot instantly get the propeller going, but the propeller speeds up steadily. The steadily increasing force between the air and the propeller is given by the equation $F = kt$, where k is a constant. If the mass of the blimp is m, find its position as a function of time. (Assume that during the period of time you're dealing with, the blimp is not yet moving fast enough to cause a significant backward force due to air resistance.) ✓ $\int$

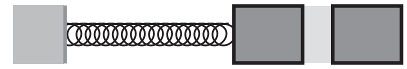
7 A car is accelerating forward along a straight road. If the force of the road on the car's wheels, pushing it forward, is a constant 3.0 kN, and the car's mass is 1000 kg, then how long will the car take to go from 20 m/s to 50 m/s? ▷ Solution, p. 278

8 Some garden shears are like a pair of scissors: one sharp blade slices past another. In the "anvil" type, however, a sharp blade presses against a flat one rather than going past it. A gardening book says that for people who are not very physically strong, the anvil type can make it easier to cut tough branches, because it concentrates the force on one side. Evaluate this claim based on Newton's laws. [Hint: Consider the forces acting on the branch, and the motion of the branch.]

9 A uranium atom deep in the earth spits out an alpha particle. An alpha particle is a fragment of an atom. This alpha particle has initial speed v, and travels a distance d before stopping in the earth.
(a) Find the force, F, that acted on the particle, in terms of v, d, and its mass, m. Don't plug in any numbers yet. Assume that the force was constant. ✓
(b) Show that your answer has the right units.
(c) Discuss how your answer to part a depends on all three variables, and show that it makes sense. That is, for each variable, discuss what would happen to the result if you changed it while keeping the other two variables constant. Would a bigger value give a smaller result, or a bigger result? Once you've figured out this *mathematical* relationship, show that it makes sense *physically*.
(d) Evaluate your result for $m = 6.7 \times 10^{-27}$ kg, $v = 2.0 \times 10^{4}$ km/s, and $d = 0.71$ mm. ✓

10 You are given a large sealed box, and are not allowed to open it. Which of the following experiments measure its mass, and which measure its weight? [Hint: Which experiments would give different results on the moon?]
(a) Put it on a frozen lake, throw a rock at it, and see how fast it scoots away after being hit.
(b) Drop it from a third-floor balcony, and measure how loud the sound is when it hits the ground.
(c) As shown in the figure, connect it with a spring to the wall, and watch it vibrate.

▷ Solution, p. 278



Problem 10, part c.

11 While escaping from the palace of the evil Martian emperor, Sally Spacehound jumps from a tower of height h down to the ground. Ordinarily the fall would be fatal, but she fires her blaster rifle straight down, producing an upward force of magnitude F_B. This force is insufficient to levitate her, but it does cancel out some of the force of gravity. During the time t that she is falling, Sally is unfortunately exposed to fire from the emperor's minions, and can't dodge their shots. Let m be her mass, and g the strength of gravity on Mars.
(a) Find the time t in terms of the other variables.
(b) Check the units of your answer to part a.
(c) For sufficiently large values of F_B, your answer to part a becomes nonsense — explain what's going on. √

12 When I cook rice, some of the dry grains always stick to the measuring cup. To get them out, I turn the measuring cup upside-down, and hit the back of the cup with my hand. Explain why this works, and why its success depends on hitting the cup hard enough.

What forces act on the girl?

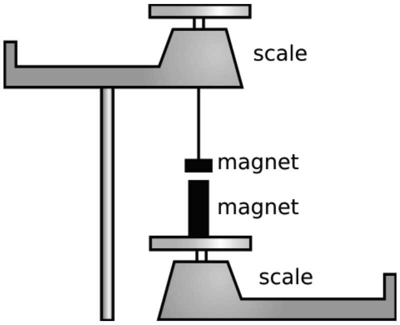
Chapter 5

Analysis of Forces

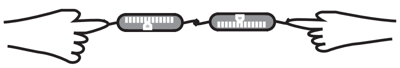
5.1 Newton's Third Law

Newton created the modern concept of force starting from his insight that all the effects that govern motion are interactions between two objects: unlike the Aristotelian theory, Newtonian physics has no phenomena in which an object changes its own motion.

Is one object always the "order-giver" and the other the "order-



a / Two magnets exert forces on each other.


b / Two people's hands exert forces on each other.


c / Rockets work by pushing exhaust gases out the back. Newton's third law says that if the rocket exerts a backward force on the gases, the gases must make an equal forward force on the rocket. Rocket engines can function above the atmosphere, unlike propellers and jets, which work by pushing against the surrounding air.

follower"? As an example, consider a batter hitting a baseball. The bat definitely exerts a large force on the ball, because the ball accelerates drastically. But if you have ever hit a baseball, you also know that the ball makes a force on the bat — often with painful results if your technique is as bad as mine!

How does the ball's force on the bat compare with the bat's force on the ball? The bat's acceleration is not as spectacular as the ball's, but maybe we shouldn't expect it to be, since the bat's mass is much greater. In fact, careful measurements of both objects' masses and accelerations would show that $m_{ball}a_{ball}$ is very nearly equal to $-m_{bat}a_{bat}$, which suggests that the ball's force on the bat is of the same magnitude as the bat's force on the ball, but in the opposite direction.

Figures a and b show two somewhat more practical laboratory experiments for investigating this issue accurately and without too much interference from extraneous forces.

In experiment a, a large magnet and a small magnet are weighed separately, and then one magnet is hung from the pan of the top balance so that it is directly above the other magnet. There is an attraction between the two magnets, causing the reading on the top scale to increase and the reading on the bottom scale to decrease. The large magnet is more "powerful" in the sense that it can pick up a heavier paperclip from the same distance, so many people have a strong expectation that one scale's reading will change by a far different amount than the other. Instead, we find that the two changes are equal in magnitude but opposite in direction: the force of the bottom magnet pulling down on the top one has the same strength as the force of the top one pulling up on the bottom one.

In experiment b, two people pull on two spring scales. Regardless of who tries to pull harder, the two forces as measured on the spring scales are equal. Interposing the two spring scales is necessary in order to measure the forces, but the outcome is not some artificial result of the scales' interactions with each other. If one person slaps another hard on the hand, the slapper's hand hurts just as much as the slappee's, and it doesn't matter if the recipient of the slap tries to be inactive. (Punching someone in the mouth causes just as much force on the fist as on the lips. It's just that the lips are more delicate. The forces are equal, but not the levels of pain and injury.)

Newton, after observing a series of results such as these, decided that there must be a fundamental law of nature at work:

Newton's third law

Forces occur in equal and opposite pairs: whenever object A exerts a force on object B, object B must also be exerting a force on object A. The two forces are equal in magnitude and opposite in direction.

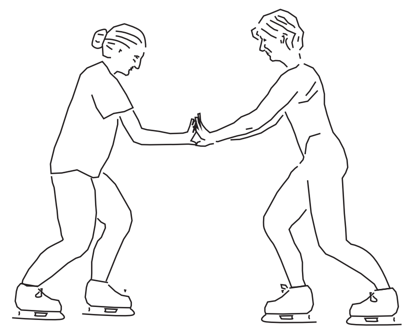
In one-dimensional situations, we can use plus and minus signs to indicate the directions of forces, and Newton's third law can be written succinctly as $F_{\text{A on B}} = -F_{\text{B on A}}$.

self-check A

Figure d analyzes swimming using Newton's third law. Do a similar analysis for a sprinter leaving the starting line. ▷ Answer, p. 272

There is no cause and effect relationship between the two forces in Newton's third law. There is no "original" force, and neither one is a response to the other. The pair of forces is a relationship, like marriage, not a back-and-forth process like a tennis match. Newton came up with the third law as a generalization about all the types of forces with which he was familiar, such as frictional and gravitational forces. When later physicists discovered a new type force, such as the force that holds atomic nuclei together, they had to check whether it obeyed Newton's third law. So far, no violation of the third law has ever been discovered, whereas the first and second laws were shown to have limitations by Einstein and the pioneers of atomic physics.

d / A swimmer doing the breast stroke pushes backward against the water. By Newton's third law, the water pushes forward on her.



e / Newton's third law does not mean that forces always cancel out so that nothing can ever move. If these two figure skaters, initially at rest, push against each other, they will both move.

The English vocabulary for describing forces is unfortunately rooted in Aristotelianism, and often implies incorrectly that forces are one-way relationships. It is unfortunate that a half-truth such as "the table exerts an upward force on the book" is so easily expressed, while a more complete and correct description ends up sounding awkward or strange: "the table and the book interact via a force," or "the table and book participate in a force."

To students, it often sounds as though Newton's third law implies nothing could ever change its motion, since the two equal and opposite forces would always cancel. The two forces, however, are always on two different objects, so it doesn't make sense to add them in the first place — we only add forces that are acting on the same object. If two objects are interacting via a force and no other forces are involved, then *both* objects will accelerate — in opposite directions!

A mnemonic for using newton's third law correctly

Mnemonics are tricks for memorizing things. For instance, the musical notes that lie between the lines on the treble clef spell the word FACE, which is easy to remember. Many people use the mnemonic "SOHCAHTOA" to remember the definitions of the sine, cosine, and tangent in trigonometry. I have my own modest offering, POFOSTITO, which I hope will make it into the mnemonics hall of fame. It's a way to avoid some of the most common problems with applying Newton's third law correctly:

f / It doesn't make sense for the man to talk about using the woman's money to cancel out his bar tab, because there is no good reason to combine his debts and her assets. Similarly, it doesn't make sense to refer to the equal and opposite forces of Newton's third law as canceling. It only makes sense to add up forces that are acting on the *same* object, whereas two forces related to each other by Newton's third law are always acting on two *different* objects.

Pair of
Opposite
Forces
Of the
Same
Type
Involving
Two
Objects

A book lying on a table *example 1*

▷ A book is lying on a table. What force is the Newton's-third-law partner of the earth's gravitational force on the book?

Answer: Newton's third law works like "B on A, A on *B*," so the partner must be the book's gravitational force pulling upward on the planet earth. Yes, there is such a force! No, it does not cause the earth to do anything noticeable.

Incorrect answer: The table's upward force on the book is the Newton's-third-law partner of the earth's gravitational force on the book.

✗This answer violates two out of three of the commandments of POFOSTITO. The forces are not of the same type, because the table's upward force on the book is not gravitational. Also, three objects are involved instead of two: the book, the table, and the planet earth.

Pushing a box up a hill *example 2*

▷ A person is pushing a box up a hill. What force is related by Newton's third law to the person's force on the box?

▷ The box's force on the person.

Incorrect answer: The person's force on the box is opposed by friction, and also by gravity.

✗This answer fails all three parts of the POFOSTITO test, the most obvious of which is that three forces are referred to instead of a pair.

▷ *Solved problem: More about example 2* *page 172, problem 20*

▷ *Solved problem: Why did it accelerate?* *page 172, problem 18*

Optional Topic: Newton's Third Law and Action at a Distance

Newton's third law is completely symmetric in the sense that neither force constitutes a delayed response to the other. Newton's third law does not even mention time, and the forces are supposed to agree at any given instant. This creates an interesting situation when it comes to noncontact forces. Suppose two people are holding magnets, and when one person waves or wiggles her magnet, the other person feels an effect on his. In this way they can send signals to each other from opposite sides of a wall, and if Newton's third law is correct, it would seem that the signals are transmitted instantly, with no time lag. The signals are indeed transmitted quite quickly, but experiments with electronically controlled magnets show that the signals do not leap the gap instantly: they travel at the same speed as light, which is an extremely high speed but not an infinite one.

Is this a contradiction to Newton's third law? Not really. According to current theories, there are no true noncontact forces. Action at a distance does not exist. Although it appears that the wiggling of one magnet affects the other with no need for anything to be in contact with anything, what really happens is that wiggling a magnet unleashes a shower of tiny particles called photons. The magnet shoves the photons out with a kick, and receives a kick in return, in strict obedience to Newton's third law. The photons fly out in all directions, and the ones that hit the other magnet then interact with it, again obeying Newton's third law.

Photons are nothing exotic, really. Light is made of photons, but our eyes receive such huge numbers of photons that we do not perceive them individually. The photons you would make by wiggling a magnet with your hand would be of a "color" that you cannot see, far off the red end of the rainbow. Book 6 in this series describes the evidence for the photon model of light.

Discussion Questions

A When you fire a gun, the exploding gases push outward in all directions, causing the bullet to accelerate down the barrel. What third-law pairs are involved? [Hint: Remember that the gases themselves are an object.]

B Tam Anh grabs Sarah by the hand and tries to pull her. She tries to remain standing without moving. A student analyzes the situation as follows. "If Tam Anh's force on Sarah is greater than her force on him, he can get her to move. Otherwise, she'll be able to stay where she is." What's wrong with this analysis?

C You hit a tennis ball against a wall. Explain any and all incorrect ideas in the following description of the physics involved: "According to Newton's third law, there has to be a force opposite to your force on the ball. The opposite force is the ball's mass, which resists acceleration, and also air resistance."

5.2 Classification and Behavior of Forces

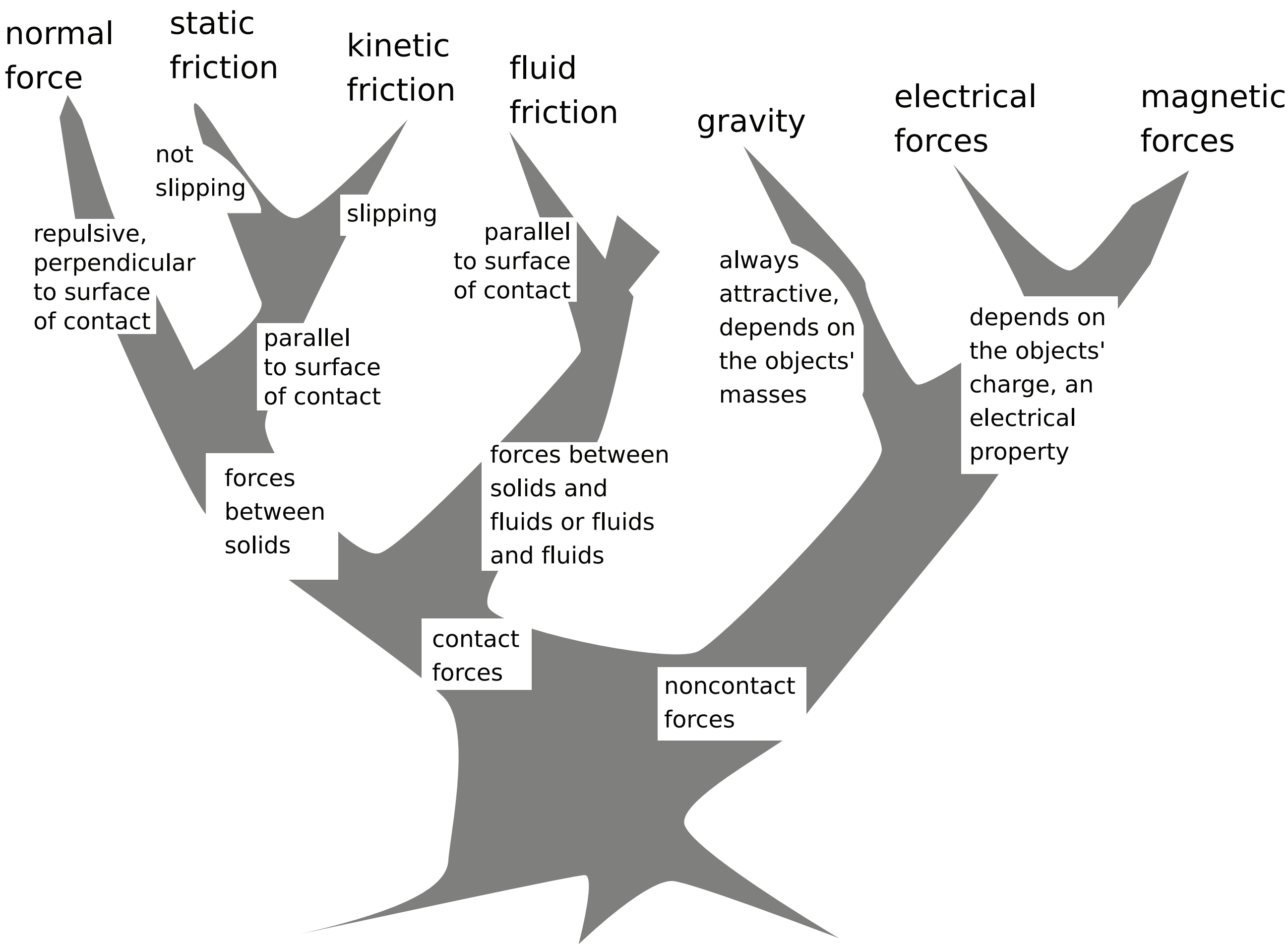
g / A scientific classification system.

One of the most basic and important tasks of physics is to classify the forces of nature. I have already referred informally to "types" of forces such as friction, magnetism, gravitational forces, and so on. Classification systems are creations of the human mind, so there is always some degree of arbitrariness in them. For one thing, the level of detail that is appropriate for a classification system depends on what you're trying to find out. Some linguists, the "lumpers," like to emphasize the similarities among languages, and a few extremists have even tried to find signs of similarities between words in languages as different as English and Chinese, lumping the world's languages into only a few large groups. Other linguists, the "splitters," might be more interested in studying the differences in pronunciation between English speakers in New York and Connecticut. The splitters call the lumpers sloppy, but the lumpers say that science isn't worthwhile unless it can find broad, simple patterns within the seemingly complex universe.

Scientific classification systems are also usually compromises between practicality and naturalness. An example is the question of how to classify flowering plants. Most people think that biological classification is about discovering new species, naming them, and classifying them in the class-order-family-genus-species system according to guidelines set long ago. In reality, the whole system is in a constant state of flux and controversy. One very practical way of classifying flowering plants is according to whether their petals are separate or joined into a tube or cone — the criterion is so clear that it can be applied to a plant seen from across the street. But here practicality conflicts with naturalness. For instance, the begonia has separate petals and the pumpkin has joined petals, but they are so similar in so many other ways that they are usually placed within the same order. Some taxonomists have come up with classification criteria that they claim correspond more naturally to the apparent relationships among plants, without having to make special exceptions, but these may be far less practical, requiring for instance the examination of pollen grains under an electron microscope.

In physics, there are two main systems of classification for forces. At this point in the course, you are going to learn one that is very practical and easy to use, and that splits the forces up into a relatively large number of types: seven very common ones that we'll discuss explicitly in this chapter, plus perhaps ten less important ones such as surface tension, which we will not bother with right now.

Physicists, however, are obsessed with finding simple patterns, so recognizing as many as fifteen or twenty types of forces strikes them as distasteful and overly complex. Since about the year 1900, physics has been on an aggressive program to discover ways in which these many seemingly different types of forces arise from a smaller number of fundamental ones. For instance, when you press your hands together, the force that keeps them from passing through each other may seem to have nothing to do with electricity, but at the atomic level, it actually does arise from electrical repulsion between atoms. By about 1950, all the forces of nature had been explained as arising from four fundamental types of forces at the atomic and nuclear level, and the lumping-together process didn't stop there. By the 1960's the length of the list had been reduced to three, and some theorists even believe that they may be able to reduce it to two or one. Although the unification of the forces of nature is one of the most beautiful and important achievements of physics, it makes much more sense to start this course with the more practical and easy system of classification. The unified system of four forces will be one of the highlights of the end of your introductory physics sequence.



h / A practical classification scheme for forces.

The practical classification scheme which concerns us now can be laid out in the form of the tree shown in figure h. The most specific types of forces are shown at the tips of the branches, and it is these types of forces that are referred to in the POFOSTITO mnemonic. For example, electrical and magnetic forces belong to the same general group, but Newton's third law would never relate an electrical force to a magnetic force.

Hitting a wall *example 3*

▷ A bullet, flying horizontally, hits a steel wall. What type of force is there between the bullet and the wall?

▷ Starting at the bottom of the tree, we determine that the force is a contact force, because it only occurs once the bullet touches the wall. Both objects are solid. The wall forms a vertical plane. If the nose of the bullet was some shape like a sphere, you might imagine that it would only touch the wall at one point. Realistically, however, we know that a lead bullet will flatten out a lot on impact, so there is a surface of contact between the two, and its orientation is vertical. The effect of the force on the bullet is to stop the horizontal motion of the bullet, and this horizontal acceleration must be produced by a horizontal force. The force is therefore perpendicular to the surface of contact, and it's also repulsive (tending to keep the bullet from entering the wall), so it must be a normal force.

The broadest distinction is that between contact and noncontact forces, which has been discussed in the previous chapter. Among the contact forces, we distinguish between those that involve solids only and those that have to do with fluids, a term used in physics to include both gases and liquids.

It should not be necessary to memorize this diagram by rote. It is better to reinforce your memory of this system by calling to mind your commonsense knowledge of certain ordinary phenomena. For instance, we know that the gravitational attraction between us and the planet earth will act even if our feet momentarily leave the ground, and that although magnets have mass and are affected by gravity, most objects that have mass are nonmagnetic.

This diagram is meant to be as simple as possible while including most of the forces we deal with in everyday life. If you were an insect, you would be much more interested in the force of surface tension, which allowed you to walk on water. I have not included the nuclear forces, which are responsible for holding the nuclei of atoms, because they are not evident in everyday life.

You should not be afraid to invent your own names for types of forces that do not fit into the diagram. For instance, the force that holds a piece of tape to the wall has been left off of the tree, and if you were analyzing a situation involving scotch tape, you would be absolutely right to refer to it by some commonsense name such as "sticky force."

On the other hand, if you are having trouble classifying a certain force, you should also consider whether it is a force at all. For instance, if someone asks you to classify the force that the earth has because of its rotation, you would have great difficulty creating a place for it on the diagram. That's because it's a type of motion, not a type of force!

Normal forces

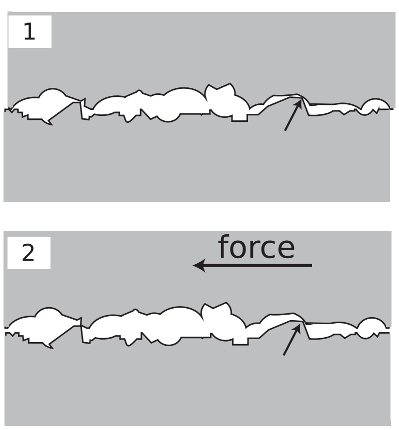
A normal force, F_N, is a force that keeps one solid object from passing through another. "Normal" is simply a fancy word for "perpendicular," meaning that the force is perpendicular to the surface of contact. Intuitively, it seems the normal force magically adjusts itself to provide whatever force is needed to keep the objects from occupying the same space. If your muscles press your hands together gently, there is a gentle normal force. Press harder, and the normal force gets stronger. How does the normal force know how strong to be? The answer is that the harder you jam your hands together, the more compressed your flesh becomes. Your flesh is acting like a spring: more force is required to compress it more. The same is true when you push on a wall. The wall flexes imperceptibly in proportion to your force on it. If you exerted enough force, would it be possible for two objects to pass through each other? No, typically the result is simply to strain the objects so much that one of them breaks.

Gravitational forces

As we'll discuss in more detail later in the course, a gravitational force exists between any two things that have mass. In everyday life, the gravitational force between two cars or two people is negligible, so the only noticeable gravitational forces are the ones between the earth and various human-scale objects. We refer to these planet-earth-induced gravitational forces as weight forces, and as we have already seen, their magnitude is given by $|F_W| = mg$.

▷ *Solved problem: Weight and mass* *page 173, problem 26*

Static and kinetic friction



i / A model that correctly explains many properties of friction. The microscopic bumps and holes in two surfaces dig into each other, causing a frictional force.

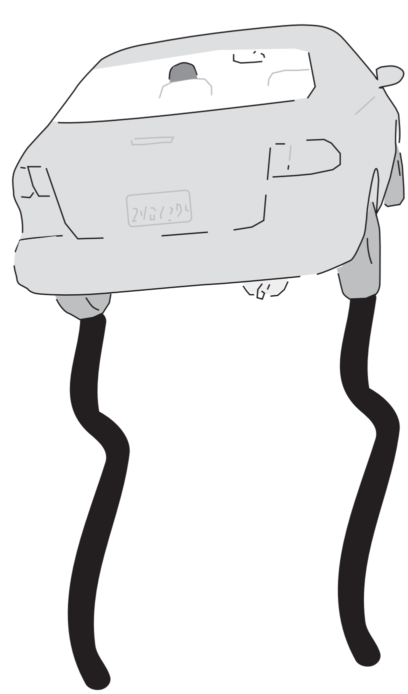
If you have pushed a refrigerator across a kitchen floor, you have felt a certain series of sensations. At first, you gradually increased your force on the refrigerator, but it didn't move. Finally, you supplied enough force to unstick the fridge, and there was a sudden jerk as the fridge started moving. Once the fridge is unstuck, you can reduce your force significantly and still keep it moving.

While you were gradually increasing your force, the floor's frictional force on the fridge increased in response. The two forces on the fridge canceled, and the fridge didn't accelerate. How did the floor know how to respond with just the right amount of force? Figure i shows one possible *model* of friction that explains this behavior. (A scientific model is a description that we expect to be incomplete, approximate, or unrealistic in some ways, but that nevertheless succeeds in explaining a variety of phenomena.) Figure i/1 shows a microscopic view of the tiny bumps and holes in the surfaces of the floor and the refrigerator. The weight of the fridge presses the two surfaces together, and some of the bumps in one surface will settle as deeply as possible into some of the holes in the other surface. In i/2, your leftward force on the fridge has caused it to ride up a little higher on the bump in the floor labeled with a small arrow. Still more force is needed to get the fridge over the bump and allow it to start moving. Of course, this is occurring simultaneously at millions of places on the two surfaces.

Once you had gotten the fridge moving at constant speed, you found that you needed to exert less force on it. Since zero total force is needed to make an object move with constant velocity, the floor's rightward frictional force on the fridge has apparently decreased somewhat, making it easier for you to cancel it out. Our model also gives a plausible explanation for this fact: as the surfaces slide past each other, they don't have time to settle down and mesh with one another, so there is less friction.

j / Static friction: the tray doesn't slip on the waiter's fingers.

Even though this model is intuitively appealing and fairly successful, it should not be taken too seriously, and in some situations it is misleading. For instance, fancy racing bikes these days are made with smooth tires that have no tread — contrary to what we'd expect from our model, this does not cause any decrease in friction. Machinists know that two very smooth and clean metal surfaces may stick to each other firmly and be very difficult to slide apart. This cannot be explained in our model, but makes more sense in terms of a model in which friction is described as arising from chemical bonds between the atoms of the two surfaces at their points of contact: very flat surfaces allow more atoms to come in contact.



k / Kinetic friction: the car skids.

Since friction changes its behavior dramatically once the surfaces come unstuck, we define two separate types of frictional forces. *Static friction* is friction that occurs between surfaces that are not slipping over each other. Slipping surfaces experience *kinetic friction.* "Kinetic" means having to do with motion. The forces of static and kinetic friction, notated F_s and F_k, are always parallel to the surface of contact between the two objects.

self-check B

1. When a baseball player slides in to a base, is the friction static, or kinetic?

2. A mattress stays on the roof of a slowly accelerating car. Is the friction static, or kinetic?

3. Does static friction create heat? Kinetic friction? ▷ Answer, p. 272

The maximum possible force of static friction depends on what kinds of surfaces they are, and also on how hard they are being pressed together. The approximate mathematical relationships can be expressed as follows:

$$F_s = -F_{\text{applied}} \quad , \quad \text{when} \quad |F_{\text{applied}}| < \mu_s |F_N| \quad ,$$

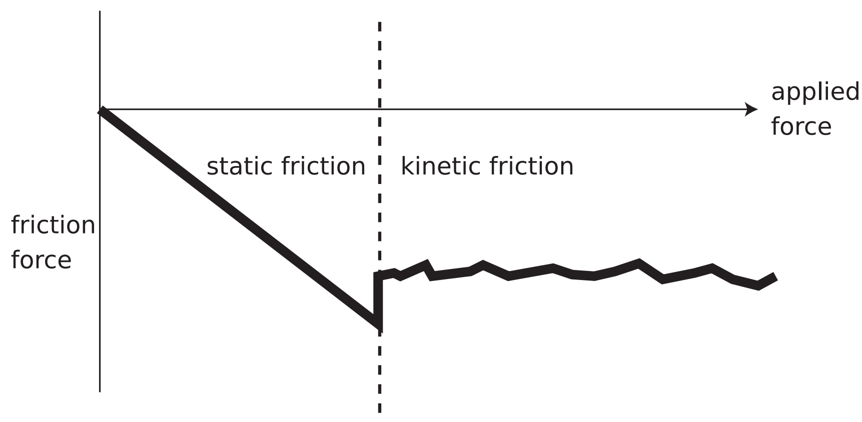
where μ_s is a unitless number, called the coefficient of static friction, which depends on what kinds of surfaces they are. The maximum force that static friction can supply, $\mu_s|F_N|$, represents the boundary between static and kinetic friction. It depends on the normal force, which is numerically equal to whatever force is pressing the two surfaces together. In terms of our model, if the two surfaces are being pressed together more firmly, a greater sideways force will be required in order to make the irregularities in the surfaces ride up and over each other.

Note that just because we use an adjective such as "applied" to refer to a force, that doesn't mean that there is some special type of force called the "applied force." The applied force could be any type of force, or it could be the sum of more than one force trying to make an object move.

The force of kinetic friction on each of the two objects is in the direction that resists the slippage of the surfaces. Its magnitude is usually well approximated as

$$|F_k| = \mu_k|F_N|$$

where μ_k is the coefficient of kinetic friction. Kinetic friction is usually more or less independent of velocity.



l / We choose a coordinate system in which the applied force, i.e., the force trying to move the objects, is positive. The friction force is then negative, since it is in the opposite direction. As you increase the applied force, the force of static friction increases to match it and cancel it out, until the maximum force of static friction is surpassed. The surfaces then begin slipping past each other, and the friction force becomes smaller in absolute value.

self-check C

Can a frictionless surface exert a normal force? Can a frictional force exist without a normal force? ▷ Answer, p. 272

If you try to accelerate or decelerate your car too quickly, the forces between your wheels and the road become too great, and they begin slipping. This is not good, because kinetic friction is weaker than static friction, resulting in less control. Also, if this occurs while you are turning, the car's handling changes abruptly because the kinetic friction force is in a different direction than the static friction force had been: contrary to the car's direction of motion, rather than contrary to the forces applied to the tire.

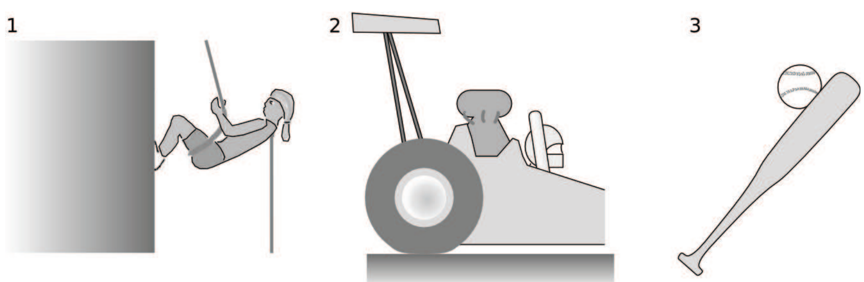
Most people respond with disbelief when told of the experimental evidence that both static and kinetic friction are approximately independent of the amount of surface area in contact. Even after doing a hands-on exercise with spring scales to show that it is true, many students are unwilling to believe their own observations, and insist that bigger tires "give more traction." In fact, the main reason why you would not want to put small tires on a big heavy car is that the tires would burst!

Although many people expect that friction would be proportional to surface area, such a proportionality would make predictions contrary to many everyday observations. A dog's feet, for example, have very little surface area in contact with the ground compared to a human's feet, and yet we know that a dog can often win a tug-of-war with a person.

The reason a smaller surface area does not lead to less friction is that the force between the two surfaces is more concentrated, causing their bumps and holes to dig into each other more deeply.

self-check D

Find the direction of each of the forces in figure m. ▷ Answer, p. 272

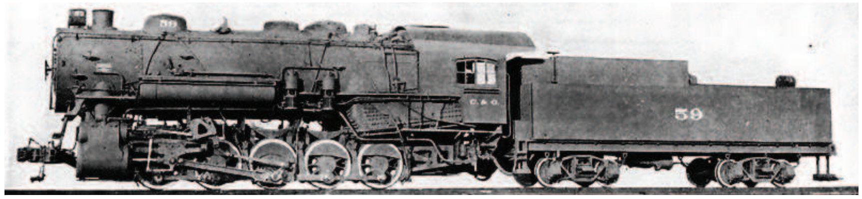


m / 1. The cliff's normal force on the climber's feet. 2. The track's static frictional force on the wheel of the accelerating dragster. 3. The ball's normal force on the bat.

Locomotives *example 4*

Looking at a picture of a locomotive, n, we notice two obvious things that are different from an automobile. Where a car typically has two drive wheels, a locomotive normally has many — ten in this example. (Some also have smaller, unpowered wheels in front of and behind the drive wheels, but this example doesn't.) Also, cars these days are generally built to be as light as possible for their size, whereas locomotives are very massive, and no effort seems to be made to keep their weight low. (The steam locomotive in the photo is from about 1900, but this is true even for modern diesel and electric trains.)

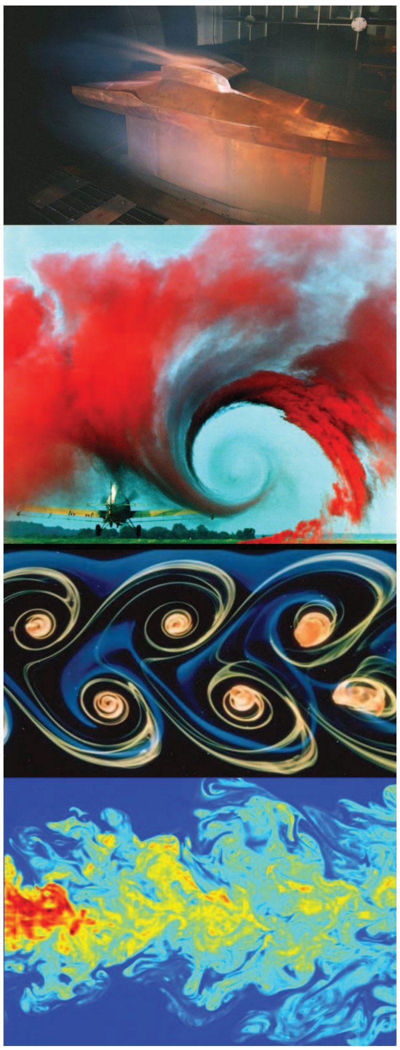
The reason locomotives are built to be so heavy is for traction. The upward normal force of the rails on the wheels, F_N, cancels the downward force of gravity, F_W, so ignoring plus and minus n / Example 4.



signs, these two forces are equal in absolute value, $F_N = F_W$. Given this amount of normal force, the maximum force of static friction is $F_s = \mu_s F_N = \mu_s F_W$. This static frictional force, of the rails pushing forward on the wheels, is the only force that can accelerate the train, pull it uphill, or cancel out the force of air resistance while cruising at constant speed. The coefficient of static friction for steel on steel is about 1/4, so no locomotive can pull with a force greater than about 1/4 of its own weight. If the engine is capable of supplying more than that amount of force, the result will be simply to break static friction and spin the wheels.

The reason this is all so different from the situation with a car is that a car isn't pulling something else. If you put extra weight in a car, you improve the traction, but you also increase the inertia of the car, and make it just as hard to accelerate. In a train, the inertia is almost all in the cars being pulled, not in the locomotive.

The other fact we have to explain is the large number of driving wheels. First, we have to realize that increasing the number of driving wheels neither increases nor decreases the total amount of static friction, because static friction is independent of the amount of surface area in contact. (The reason four-wheel-drive is good in a car is that if one or more of the wheels is slipping on ice or in mud, the other wheels may still have traction. This isn't typically an issue for a train, since all the wheels experience the same conditions.) The advantage of having more driving wheels on a train is that it allows us to increase the weight of the locomotive without crushing the rails, or damaging bridges.



o / Fluid friction depends on the fluid's pattern of flow, so it is more complicated than friction between solids, and there are no simple, universally applicable formulas to calculate it. From top to bottom: supersonic wind tunnel, vortex created by a crop duster, series of vortices created by a single object, turbulence.

Fluid friction

Try to drive a nail into a waterfall and you will be confronted with the main difference between solid friction and fluid friction. Fluid friction is purely kinetic; there is no static fluid friction. The nail in the waterfall may tend to get dragged along by the water flowing past it, but it does not stick in the water. The same is true for gases such as air: recall that we are using the word "fluid" to include both gases and liquids.

Unlike kinetic friction between solids, fluid friction increases rapidly with velocity. It also depends on the shape of the object, which is why a fighter jet is more streamlined than a Model T. For objects of the same shape but different sizes, fluid friction typically scales up with the cross-sectional area of the object, which is one of the main reasons that an SUV gets worse mileage on the freeway than a compact car.

p / What do the golf ball and the shark have in common? Both use the same trick to reduce fluid friction. The dimples on the golf ball modify the pattern of flow of the air around it, counterintuitively *reducing* friction. Recent studies have shown that sharks can accomplish the same thing by raising, or "bristling," the scales on their skin at high speeds.

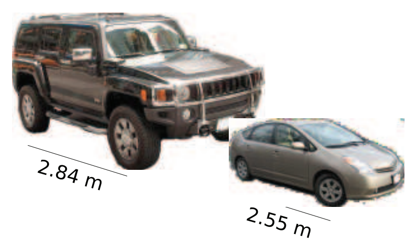
Discussion Questions

A A student states that when he tries to push his refrigerator, the reason it won't move is because Newton's third law says there's an equal and opposite frictional force pushing back. After all, the static friction force is equal and opposite to the applied force. How would you convince him he is wrong?

B Kinetic friction is usually more or less independent of velocity. However, inexperienced drivers tend to produce a jerk at the last moment of deceleration when they stop at a stop light. What does this tell you about the kinetic friction between the brake shoes and the brake drums?

C Some of the following are correct descriptions of types of forces that could be added on as new branches of the classification tree. Others are not really types of forces, and still others are not force phenomena at all. In each case, decide what's going on, and if appropriate, figure out how you would incorporate them into the tree.

sticky force	makes tape stick to things
opposite force	the force that Newton's third law says relates to every force you make
flowing force	the force that water carries with it as it flows out of a hose
surface tension	lets insects walk on water
horizontal force	a force that is horizontal
motor force	the force that a motor makes on the thing it is turning
canceled force	a force that is being canceled out by some other force



q / The wheelbases of the Hummer H3 and the Toyota Prius are surprisingly similar, differing by only 10%. The main difference in shape is that the Hummer is much taller and wider. It presents a much greater cross-sectional area to the wind, and this is the main reason that it uses about 2.5 times more gas on the freeway.

5.3 Analysis of Forces

Newton's first and second laws deal with the total of all the forces exerted on a specific object, so it is very important to be able to figure out what forces there are. Once you have focused your attention on one object and listed the forces on it, it is also helpful to describe all the corresponding forces that must exist according to Newton's third law. We refer to this as "analyzing the forces" in which the object participates.

A barge *example 5*

A barge is being pulled along a canal by teams of horses on the shores. Analyze all the forces in which the barge participates.

force acting on barge	*force related to it by Newton's third law*
ropes' forward normal forces on barge	barge's backward normal force on ropes
water's backward fluid friction force on barge	barge's forward fluid friction force on water
planet earth's downward gravitational force on barge	barge's upward gravitational force on earth
water's upward "floating" force on barge	barge's downward "floating" force on water

Here I've used the word "floating" force as an example of a sensible invented term for a type of force not classified on the tree in the previous section. A more formal technical term would be "hydrostatic force."

Note how the pairs of forces are all structured as "A's force on B, B's force on A": ropes on barge and barge on ropes; water on barge and barge on water. Because all the forces in the left column are forces acting on the barge, all the forces in the right column are forces being exerted by the barge, which is why each entry in the column begins with "barge."

Often you may be unsure whether you have forgotten one of the forces. Here are three strategies for checking your list:

> See what physical result would come from the forces you've found so far. Suppose, for instance, that you'd forgotten the "floating" force on the barge in the example above. Looking at the forces you'd found, you would have found that there was a downward gravitational force on the barge which was not canceled by any upward force. The barge isn't supposed to sink, so you know you need to find a fourth, upward force.
>
> Another technique for finding missing forces is simply to go through the list of all the common types of forces and see if any of them apply.
>
> Make a drawing of the object, and draw a dashed boundary line around it that separates it from its environment. Look for points on the boundary where other objects come in contact with your object. This strategy guarantees that you'll find every contact force that acts on the object, although it won't help you to find non-contact forces.

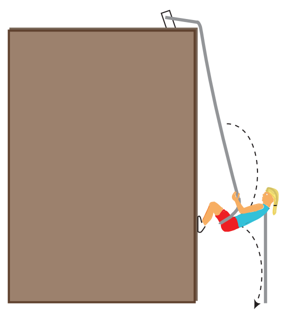
The following is another example in which we can profit by checking against our physical intuition for what should be happening.

Rappelling *example 6*

As shown in the figure below, Cindy is rappelling down a cliff. Her downward motion is at constant speed, and she takes little hops off of the cliff, as shown by the dashed line. Analyze the forces in which she participates at a moment when her feet are on the cliff and she is pushing off.

force acting on Cindy	*force related to it by Newton's third law*
planet earth's downward gravitational force on Cindy	Cindy's upward gravitational force on earth
ropes upward frictional force on Cindy (her hand)	Cindy's downward frictional force on the rope
cliff's rightward normal force on Cindy	Cindy's leftward normal force on the cliff

The two vertical forces cancel, which is what they should be doing if she is to go down at a constant rate. The only horizontal force on her is the cliff's force, which is not canceled by any other force, and which therefore will produce an acceleration of Cindy to the right. This makes sense, since she is hopping off. (This solution is a little oversimplified, because the rope is slanting, so it also applies a small leftward force to Cindy. As she flies out to the right, the slant of the rope will increase, pulling her back in more strongly.)



I believe that constructing the type of table described in this section is the best method for beginning students. Most textbooks, however, prescribe a pictorial way of showing all the forces acting on an object. Such a picture is called a free-body diagram. It should not be a big problem if a future physics professor expects you to be able to draw such diagrams, because the conceptual reasoning is the same. You simply draw a picture of the object, with arrows representing the forces that are acting on it. Arrows representing contact forces are drawn from the point of contact, noncontact forces from the center of mass. Free-body diagrams do not show the equal and opposite forces exerted by the object itself.

Often you may be unsure whether you have missed one of the forces. Here are three strategies for checking your list:

> See what physical result would come from the forces you've found so far. Suppose, for instance, that you'd forgotten the "floating" force on the barge in the example above. Looking at the forces you'd found, you would have found that there was a downward gravitational force on the barge which was not canceled by any upward force. The barge isn't supposed to sink, so you know you need to find a fourth, upward force.
>
> Whenever one solid object touches another, there will be a normal force, and possibly also a frictional force; check for both.

Make a drawing of the object, and draw a dashed boundary line around it that separates it from its environment. Look for points on the boundary where other objects come in contact with your object. This strategy guarantees that you'll find every contact force that acts on the object, although it won't help you to find non-contact forces.

Discussion question C.

Discussion Questions

A In the example of the barge going down the canal, I referred to a "floating" or "hydrostatic" force that keeps the boat from sinking. If you were adding a new branch on the force-classification tree to represent this force, where would it go?

B A pool ball is rebounding from the side of the pool table. Analyze the forces in which the ball participates during the short time when it is in contact with the side of the table.

C The earth's gravitational force on you, i.e., your weight, is always equal to mg, where m is your mass. So why can you get a shovel to go deeper into the ground by jumping onto it? Just because you're jumping, that doesn't mean your mass or weight is any greater, does it?

5.4 Transmission of Forces by Low-Mass Objects

You're walking your dog. The dog wants to go faster than you do, and the leash is taut. Does Newton's third law guarantee that your force on your end of the leash is equal and opposite to the dog's force on its end? If they're not exactly equal, is there any reason why they should be approximately equal?

If there was no leash between you, and you were in direct contact with the dog, then Newton's third law would apply, but Newton's third law cannot relate your force on the leash to the dog's force on the leash, because that would involve three separate objects. Newton's third law only says that your force on the leash is equal and opposite to the leash's force on you,

$$F_{yL} = -F_{Ly},$$

and that the dog's force on the leash is equal and opposite to its force on the dog

$$F_{dL} = -F_{Ld}.$$

Still, we have a strong intuitive expectation that whatever force we make on our end of the leash is transmitted to the dog, and vice-versa. We can analyze the situation by concentrating on the forces that act on the leash, F_{dL} and F_{yL}. According to Newton's second law, these relate to the leash's mass and acceleration:

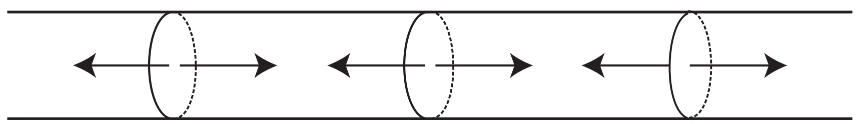
$$F_{dL} + F_{yL} = m_L a_L.$$

The leash is far less massive then any of the other objects involved, and if m_L is very small, then apparently the total force on the leash is also very small, $F_{dL} + F_{yL} \approx 0$, and therefore

$$F_{dL} \approx -F_{yL} \qquad .$$

Thus even though Newton's third law does not apply directly to these two forces, we can approximate the low-mass leash as if it was not intervening between you and the dog. It's at least approximately as if you and the dog were acting directly on each other, in which case Newton's third law would have applied.

In general, low-mass objects can be treated approximately as if they simply transmitted forces from one object to another. This can be true for strings, ropes, and cords, and also for rigid objects such as rods and sticks.



r / If we imagine dividing a taut rope up into small segments, then any segment has forces pulling outward on it at each end. If the rope is of negligible mass, then all the forces equal $+T$ or $-T$, where T, the tension, is a single number.

If you look at a piece of string under a magnifying glass as you pull on the ends more and more strongly, you will see the fibers straightening and becoming taut. Different parts of the string are apparently exerting forces on each other. For instance, if we think of the two halves of the string as two objects, then each half is exerting a force on the other half. If we imagine the string as consisting of many small parts, then each segment is transmitting a force to the next segment, and if the string has very little mass, then all the forces are equal in magnitude. We refer to the magnitude of the forces as the tension in the string, T. Although the tension is measured in units of Newtons, it is not itself a force. There are many forces within the string, some in one direction and some in the other direction, and their magnitudes are only approximately equal. The concept of tension only makes sense as a general, approximate statement of how big all the forces are.

s / The Golden Gate Bridge's roadway is held up by the tension in the vertical cables.

If a rope goes over a pulley or around some other object, then the tension throughout the rope is approximately equal so long as there is not too much friction. A rod or stick can be treated in much the same way as a string, but it is possible to have either compression or tension.

Since tension is not a type of force, the force exerted by a rope on some other object must be of some definite type such as static friction, kinetic friction, or a normal force. If you hold your dog's leash with your hand through the loop, then the force exerted by the leash on your hand is a normal force: it is the force that keeps the leash from occupying the same space as your hand. If you grasp a plain end of a rope, then the force between the rope and your hand is a frictional force.

A more complex example of transmission of forces is the way a car accelerates. Many people would describe the car's engine as making the force that accelerates the car, but the engine is part of the car, so that's impossible: objects can't make forces on themselves. What really happens is that the engine's force is transmitted through the transmission to the axles, then through the tires to the road. By Newton's third law, there will thus be a forward force from the road on the tires, which accelerates the car.

Discussion Question

A When you step on the gas pedal, is your foot's force being transmitted in the sense of the word used in this section?

5.5 Objects Under Strain

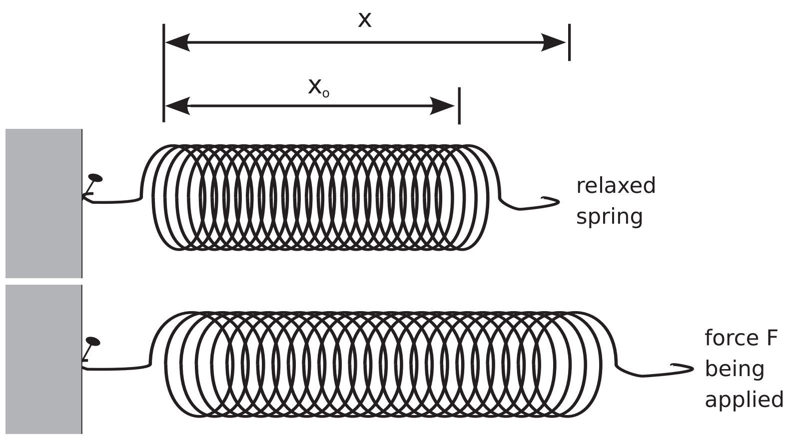
A string lengthens slightly when you stretch it. Similarly, we have already discussed how an apparently rigid object such as a wall is actually flexing when it participates in a normal force. In other cases, the effect is more obvious. A spring or a rubber band visibly elongates when stretched.

Common to all these examples is a change in shape of some kind: lengthening, bending, compressing, etc. The change in shape can be measured by picking some part of the object and measuring its position, x. For concreteness, let's imagine a spring with one end attached to a wall. When no force is exerted, the unfixed end of the spring is at some position x_o. If a force acts at the unfixed end, its position will change to some new value of x. The more force, the greater the departure of x from x_o.

Back in Newton's time, experiments like this were considered cutting-edge research, and his contemporary Hooke is remembered today for doing them and for coming up with a simple mathematical generalization called Hooke's law:

$$F \approx k(x - x_o) \qquad . \qquad \text{[force required to stretch a spring; valid for small forces only]}$$

Here k is a constant, called the spring constant, that depends on how stiff the object is. If too much force is applied, the spring exhibits more complicated behavior, so the equation is only a good approximation if the force is sufficiently small. Usually when the force is so large that Hooke's law is a bad approximation, the force ends up permanently bending or breaking the spring.



t / Defining the quantities F, x, and x_o in Hooke's law.

Although Hooke's law may seem like a piece of trivia about springs, it is actually far more important than that, because all solid objects exert Hooke's-law behavior over some range of sufficiently small forces. For example, if you push down on the hood of a car, it dips by an amount that is directly proportional to the force. (But the car's behavior would not be as mathematically simple if you dropped a boulder on the hood!)

▷ *Solved problem: Combining springs* *page 171, problem 14*

▷ *Solved problem: Young's modulus* *page 171, problem 16*

Discussion Question

A A car is connected to its axles through big, stiff springs called shock absorbers, or "shocks." Although we've discussed Hooke's law above only in the case of stretching a spring, a car's shocks are continually going through both stretching and compression. In this situation, how would you interpret the positive and negative signs in Hooke's law?

5.6 Simple Machines: the Pulley

Even the most complex machines, such as cars or pianos, are built out of certain basic units called *simple machines.* The following are some of the main functions of simple machines:

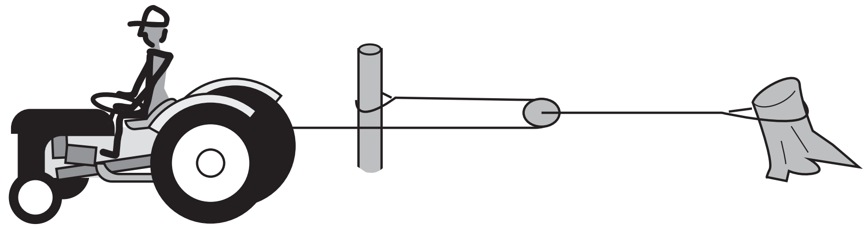
transmitting a force: The chain on a bicycle transmits a force from the crank set to the rear wheel.

changing the direction of a force: If you push down on a seesaw, the other end goes up.

changing the speed and precision of motion: When you make the "come here" motion, your biceps only moves a couple of centimeters where it attaches to your forearm, but your arm moves much farther and more rapidly.

changing the amount of force: A lever or pulley can be used to increase or decrease the amount of force.

You are now prepared to understand one-dimensional simple machines, of which the pulley is the main example.



u / Example 7.

A pulley *example 7*

▷ Farmer Bill says this pulley arrangement doubles the force of his tractor. Is he just a dumb hayseed, or does he know what he's doing?

▷ To use Newton's first law, we need to pick an object and consider the sum of the forces on it. Since our goal is to relate the tension in the part of the cable attached to the stump to the tension in the part attached to the tractor, we should pick an object to which both those cables are attached, i.e., the pulley itself. As discussed in section 5.4, the tension in a string or cable remains approximately constant as it passes around a pulley, provided that there is not too much friction. There are therefore two leftward forces acting on the pulley, each equal to the force exerted by the tractor. Since the acceleration of the pulley is essentially zero, the forces on it must be canceling out, so the rightward force of the pulley-stump cable on the pulley must be double the force exerted by the tractor. Yes, Farmer Bill knows what he's talking about.

Summary

Selected Vocabulary

repulsive	describes a force that tends to push the two participating objects apart
attractive	describes a force that tends to pull the two participating objects together
oblique	describes a force that acts at some other angle, one that is not a direct repulsion or attraction
normal force . . .	the force that keeps two objects from occupying the same space
static friction . .	a friction force between surfaces that are not slipping past each other
kinetic friction .	a friction force between surfaces that are slipping past each other
fluid	a gas or a liquid
fluid friction . . .	a friction force in which at least one of the object is is a fluid
spring constant .	the constant of proportionality between force and elongation of a spring or other object under strain

Notation

F_N	a normal force
F_s	a static frictional force
F_k	a kinetic frictional force
μ_s	the coefficient of static friction; the constant of proportionality between the maximum static frictional force and the normal force; depends on what types of surfaces are involved
μ_k	the coefficient of kinetic friction; the constant of proportionality between the kinetic frictional force and the normal force; depends on what types of surfaces are involved
k	the spring constant; the constant of proportionality between the force exerted on an object and the amount by which the object is lengthened or compressed

Summary

Newton's third law states that forces occur in equal and opposite pairs. If object A exerts a force on object B, then object B must simultaneously be exerting an equal and opposite force on object A. Each instance of Newton's third law involves exactly two objects, and exactly two forces, which are of the same type.

There are two systems for classifying forces. We are presently using the more practical but less fundamental one. In this system, forces are classified by whether they are repulsive, attractive, or oblique; whether they are contact or noncontact forces; and whether the two objects involved are solids or fluids.

Static friction adjusts itself to match the force that is trying to make the surfaces slide past each other, until the maximum value is reached,

$$|F_s| < \mu_s|F_N| \qquad .$$

Once this force is exceeded, the surfaces slip past one another, and kinetic friction applies,

$$|F_k| = \mu_k|F_N| \qquad .$$

Both types of frictional force are nearly independent of surface area, and kinetic friction is usually approximately independent of the speed at which the surfaces are slipping.

A good first step in applying Newton's laws of motion to any physical situation is to pick an object of interest, and then to list all the forces acting on that object. We classify each force by its type, and find its Newton's-third-law partner, which is exerted by the object on some other object.

When two objects are connected by a third low-mass object, their forces are transmitted to each other nearly unchanged.

Objects under strain always obey Hooke's law to a good approximation, as long as the force is small. Hooke's law states that the stretching or compression of the object is proportional to the force exerted on it,

$$F \approx k(x - x_o) \qquad .$$

Problems

Key

- √ A computerized answer check is available online.
- ∫ A problem that requires calculus.
- ⋆ A difficult problem.

1 A little old lady and a pro football player collide head-on. Compare their forces on each other, and compare their accelerations. Explain.

Problem 1.

2 The earth is attracted to an object with a force equal and opposite to the force of the earth on the object. If this is true, why is it that when you drop an object, the earth does not have an acceleration equal and opposite to that of the object?

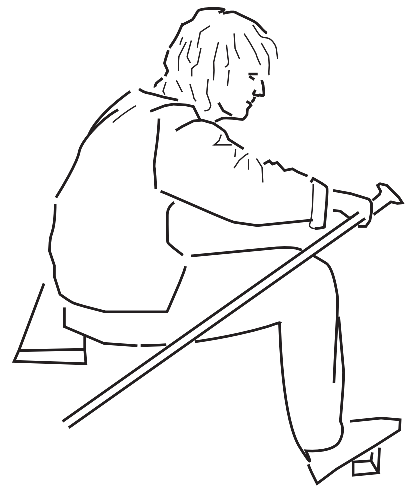
3 When you stand still, there are two forces acting on you, the force of gravity (your weight) and the normal force of the floor pushing up on your feet. Are these forces equal and opposite? Does Newton's third law relate them to each other? Explain.

In problems 4-8, analyze the forces using a table in the format shown in section 5.3. Analyze the forces in which the italicized object participates.

4 A *magnet* is stuck underneath a parked car. (See instructions above.)

5 Analyze two examples of *objects* at rest relative to the earth that are being kept from falling by forces other than the normal force. Do not use objects in outer space, and do not duplicate problem 4 or 8. (See instructions above.)

6 A *person* is rowing a boat, with her feet braced. She is doing the part of the stroke that propels the boat, with the ends of the oars in the water (not the part where the oars are out of the water). (See instructions above.)

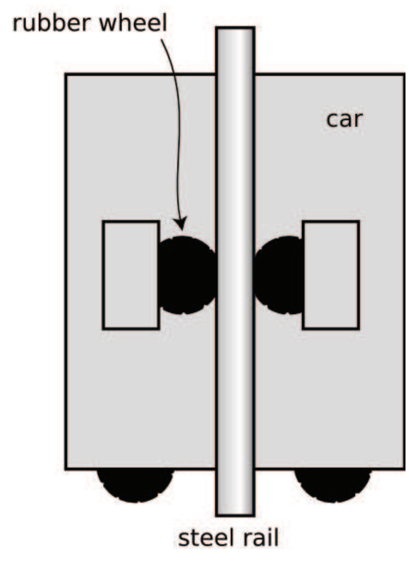


Problem 6.

7 A *farmer* is in a stall with a cow when the cow decides to press him against the wall, pinning him with his feet off the ground. Analyze the forces in which the farmer participates. (See instructions above.)

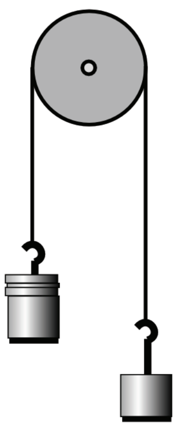
8 A propeller *plane* is cruising east at constant speed and altitude. (See instructions above.)

9 Today's tallest buildings are really not that much taller than the tallest buildings of the 1940*s*. One big problem with making an even taller skyscraper is that every elevator needs its own shaft running the whole height of the building. So many elevators are needed to serve the building's thousands of occupants that the elevator shafts start taking up too much of the space within the building. An alternative is to have elevators that can move both horizontally and vertically: with such a design, many elevator cars can share a



Problem 9.

few shafts, and they don't get in each other's way too much because they can detour around each other. In this design, it becomes impossible to hang the cars from cables, so they would instead have to ride on rails which they grab onto with wheels. Friction would keep them from slipping. The figure shows such a frictional elevator in its vertical travel mode. (The wheels on the bottom are for when it needs to switch to horizontal motion.)
(a) If the coefficient of static friction between rubber and steel is μ_s, and the maximum mass of the car plus its passengers is M, how much force must there be pressing each wheel against the rail in order to keep the car from slipping? (Assume the car is not accelerating.) ✓
(b) Show that your result has physically reasonable behavior with respect to μ_s. In other words, if there was less friction, would the wheels need to be pressed more firmly or less firmly? Does your equation behave that way?



Problem 10.

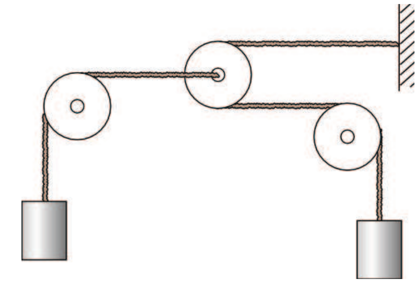
10 Unequal masses M and m are suspended from a pulley as shown in the figure.
(a) Analyze the forces in which mass m participates, using a table the format shown in section 5.3. [The forces in which the other mass participates will of course be similar, but not numerically the same.]
(b) Find the magnitude of the accelerations of the two masses. [Hints: (1) Pick a coordinate system, and use positive and negative signs consistently to indicate the directions of the forces and accelerations. (2) The two accelerations of the two masses have to be equal in magnitude but of opposite signs, since one side eats up rope at the same rate at which the other side pays it out. (3) You need to apply Newton's second law twice, once to each mass, and then solve the two equations for the unknowns: the acceleration, a, and the tension in the rope, T.]
(c) Many people expect that in the special case of $M = m$, the two masses will naturally settle down to an equilibrium position side by side. Based on your answer from part b, is this correct?
(d) Find the tension in the rope, T.
(e) Interpret your equation from part d in the special case where one of the masses is zero. Here "interpret" means to figure out what happens mathematically, figure out what should happen physically, and connect the two.

11 A tugboat of mass m pulls a ship of mass M, accelerating it. The speeds are low enough that you can ignore fluid friction acting on their hulls, although there will of course need to be fluid friction acting on the tug's propellers.
(a) Analyze the forces in which the tugboat participates, using a table in the format shown in section 5.3. Don't worry about vertical forces.
(b) Do the same for the ship.

(c) Assume now that water friction on the two vessels' hulls is negligible. If the force acting on the tug's propeller is F, what is the tension, T, in the cable connecting the two ships? [Hint: Write down two equations, one for Newton's second law applied to each object. Solve these for the two unknowns T and a.] √
(d) Interpret your answer in the special cases of $M = 0$ and $M = \infty$.

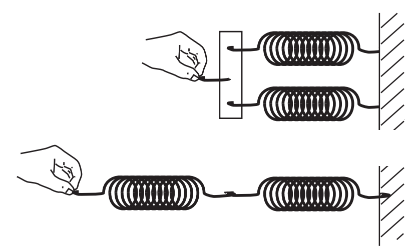
12 Someone tells you she knows of a certain type of Central American earthworm whose skin, when rubbed on polished diamond, has $\mu_k > \mu_s$. Why is this not just empirically unlikely but logically suspect?

13 In the system shown in the figure, the pulleys on the left and right are fixed, but the pulley in the center can move to the left or right. The two masses are identical. Show that the mass on the left will have an upward acceleration equal to $g/5$. Assume all the ropes and pulleys are massless and rictionless.



Problem 13.

14 The figure shows two different ways of combining a pair of identical springs, each with spring constant k. We refer to the top setup as parallel, and the bottom one as a series arrangement.
(a) For the parallel arrangement, analyze the forces acting on the connector piece on the left, and then use this analysis to determine the equivalent spring constant of the whole setup. Explain whether the combined spring constant should be interpreted as being stiffer or less stiff.
(b) For the series arrangement, analyze the forces acting on each spring and figure out the same things. ▷ Solution, p. 278

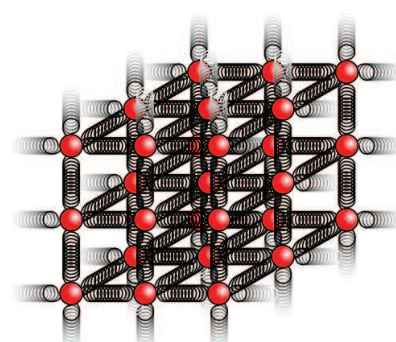


Problem 14.

15 Generalize the results of problem 14 to the case where the two spring constants are unequal.

16 (a) Using the solution of problem 14, which is given in the back of the book, predict how the spring constant of a fiber will depend on its length and cross-sectional area.
(b) The constant of proportionality is called the Young's modulus, E, and typical values of the Young's modulus are about 10^{10} to 10^{11}. What units would the Young's modulus have in the SI (meter-kilogram-second) system? ▷ Solution, p. 279

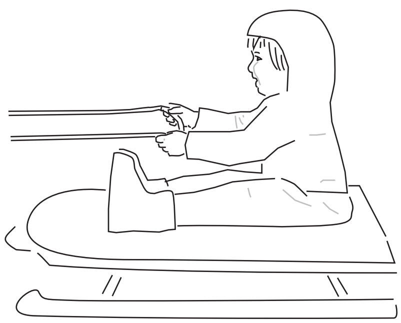
17 This problem depends on the results of problems 14 and 16, whose solutions are in the back of the book. When atoms form chemical bonds, it makes sense to talk about the spring constant of the bond as a measure of how "stiff" it is. Of course, there aren't really little springs — this is just a mechanical model. The purpose of this problem is to estimate the spring constant, k, for a single bond in a typical piece of solid matter. Suppose we have a fiber, like a hair or a piece of fishing line, and imagine for simplicity that it is made of atoms of a single element stacked in a cubical manner, as shown in the figure, with a center-to-center spacing b. A typical value for b would be about 10^{-10} m.



Problem 17.

(a) Find an equation for k in terms of b, and in terms of the Young's modulus, E, defined in problem 16 and its solution.
(b) Estimate k using the numerical data given in problem 16.
(c) Suppose you could grab one of the atoms in a diatomic molecule like H_2 or O_2, and let the other atom hang vertically below it. Does the bond stretch by any appreciable fraction due to gravity?

18 In each case, identify the force that causes the acceleration, and give its Newton's-third-law partner. Describe the effect of the partner force. (a) A swimmer speeds up. (b) A golfer hits the ball off of the tee. (c) An archer fires an arrow. (d) A locomotive slows down. ▷ Solution, p. 279


Problem 19.

19 Ginny has a plan. She is going to ride her sled while her dog Foo pulls her, and she holds on to his leash. However, Ginny hasn't taken physics, so there may be a problem: she may slide right off the sled when Foo starts pulling.
(a) Analyze all the forces in which Ginny participates, making a table as in section 5.3.
(b) Analyze all the forces in which the sled participates.
(c) The sled has mass m, and Ginny has mass M. The coefficient of static friction between the sled and the snow is μ_1, and μ_2 is the corresponding quantity for static friction between the sled and her snow pants. Ginny must have a certain minimum mass so that she will not slip off the sled. Find this in terms of the other three variables. ✓
(d) Interpreting your equation from part c, under what conditions will there be no physically realistic solution for M? Discuss what this means physically.

20 Example 2 on page 148 involves a person pushing a box up a hill. The incorrect answer describes three forces. For each of these three forces, give the force that it is related to by Newton's third law, and state the type of force. ▷ Solution, p. 279

21 Example 7 on page 166 describes a force-doubling setup involving a pulley. Make up a more complicated arrangement, using more than one pulley, that would multiply the force by a factor greater than two.

22 Pick up a heavy object such as a backpack or a chair, and stand on a bathroom scale. Shake the object up and down. What do you observe? Interpret your observations in terms of Newton's third law.

23 A cop investigating the scene of an accident measures the length L of a car's skid marks in order to find out its speed v at the beginning of the skid. Express v in terms of L and any other relevant variables. ✓

24 The following reasoning leads to an apparent paradox; explain what's wrong with the logic. A baseball player hits a ball. The ball and the bat spend a fraction of a second in contact. During that time they're moving together, so their accelerations must be equal. Newton's third law says that their forces on each other are also equal. But $a = F/m$, so how can this be, since their masses are unequal? (Note that the paradox isn't resolved by considering the force of the batter's hands on the bat. Not only is this force very small compared to the ball-bat force, but the batter could have just thrown the bat at the ball.)

25 This problem has been deleted.

26 (a) Compare the mass of a one-liter water bottle on earth, on the moon, and in interstellar space. ▷ Solution, p. 279
(b) Do the same for its weight.

27 An ice skater builds up some speed, and then coasts across the ice passively in a straight line. (a) Analyze the forces.
(b) If his initial speed is v, and the coefficient of kinetic friction is μ_k, find the maximum theoretical distance he can glide before coming to a stop. Ignore air resistance. √
(c) Show that your answer to part b has the right units.
(d) Show that your answer to part b depends on the variables in a way that makes sense physically.
(e) Evaluate your answer numerically for $\mu_k = 0.0046$, and a world-record speed of 14.58 m/s. (The coefficient of friction was measured by De Koning et al., using special skates worn by real speed skaters.) √
(f) Comment on whether your answer in part e seems realistic. If it doesn't, suggest possible reasons why.